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Applied Mechanics Reviews

A Critical Review of the World Literature in Applied Mechanics

A. W. WUNDHEILER, *Editor*

T. VON KÁRMÁN, S. TIMOSHENKO, *Editorial Advisers*

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v	g	e	yo	zh	i	ĭ	u	kh	ts	ch	sh	shch	″	ĭ	′	è	yu	ya

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Applied Mechanics Reviews

A Critical Review of the World Literature in Applied Mechanics

February 1950 Vol. 3, No. 2

Theoretical and Experimental Methods

(See also Revs. 209, 211, 212, 216, 217, 232, 294)

195. American Men of Science, a biographical directory (Jacques Cattell, Editor), Lancaster, Pa., The Science Press, 1949, 2836 pp. Cloth, 9.7 × 7.3 in., $17.

This eighth edition contains about 50,000 biographies, including about 20,000 names that have not appeared in earlier editions.

Ed.

196. Arnold Sommerfeld, Partial differential equations in physics (translated by Ernst G. Straus), Academic Press, Inc., New York, 1949, 335 pp. Cloth, 6 × 9 in., 42 figs., $5.80.

This book is subdivided into the following six chapters: I. Fourier series and integrals, 32 pp. II. Introduction to partial differential equations, 32 pp. III. Boundary value problems in heat, 22 pp. IV. Cylinder and sphere problems (Bessel and Hankel functions, spherical harmonics, etc.), 83 pp. V. Eigenfunctions and eigenvalues, 71 pp. VI. Problems of radio, 55 pp. Exercises (42 pp.) grouped according to the six chapters and followed by hints complete the book.

The work corresponds with the concluding part of the author's six-term course in physics, and deals, in his words, with physical mathematics. Didactic aims are uppermost in his mind, and the formal mathematical part is interrupted by typical examples of applications.

Ed.

197. Albert Betz, Conformal mapping (Konforme Abbildung), Berlin, Göttingen, Heidelberg, Springer-Verlag, 1948, viii + 359 pp. 6.5 × 9.5 in., 276 figs., DM 36 (approx. $10).

To the mathematician, conformal mapping is a chapter in the theory of functions; to the engineer and physical scientist, conformal mapping is a working tool for the solution of practical problems. Many books are available which discuss the subject from the point of view of the theory of functions of complex variables. This book is written for those whose principal interest is in the practical use of conformal mapping, especially for engineers to whom the abstract ideas of function theory are somewhat unfamiliar. The author's aim is to give an introduction to the geometrical relations of conformal maps and a guide to the practical use of this powerful tool.

The book originated in some lectures given by Betz in 1932, in which this approach was tried. Later Irmgard Lotz, Mangler, and Krohn collaborated in revising and expanding the treatment to appeal also to students with a mathematical background. Thus the book proceeds from the geometrical aspects of mapping to applications of conformal mapping to electric-current fields in two dimensions, and to applications to other fields of physics, and thence to the theory of complex functions and their application to conformal mapping.

The eleven chapter headings are as follows: Introduction and simple examples, Electric current fields, Further examples and consequences, General considerations, Occurrence of conformal mapping in other fields of physics, Relation of conformal mapping to the theory of complex functions, Mapping by simple functions, Some collected functions, Treatment of given mapping problems, Doubly-periodic fields, Free jets.

In an appendix, the author lists the general methods for solving problems in conformal mapping which are treated briefly in the text, namely computational methods, graphical methods, and experimental methods by means of electric-current fields, by means of the Hele Shaw apparatus with two-dimensional viscous flow of a fluid, and by means of elastic membranes. There are also listed about 70 mapping functions with sketches of the contours in the two mapping planes and references to the discussion in the text

The author purposely omits a detailed bibliography, confining the references to more extended treatments of matters discussed in the book.

Hugh L. Dryden, USA

198. H. Poritsky, Homogeneous harmonic functions, Quart. appl. Math. 6, 379–388 (Jan. 1949).

The author investigates homogeneous solutions φ_n of the degree n of the equation $\varphi_{xx} + \varphi_{yy} + \varphi_{zz} = 0$. Using two alternative methods he shows at first that the solutions φ_0 can be written in the form $\varphi_0 = \mathrm{Re}\,[f(se^{i\omega})]$, where $s = \log\,[r/(z + (z^2 + R^2)^{1/2})]$. Here f is an analytic function of a complex variable, z, r, ω are cylindrical coordinates, $R = (x^2 + y^2 + z^2)^{1/2}$. It is further shown that φ_n for $n > 0$ are polynomials in x,y,z with coefficients which are harmonic functions of the degree zero, and that it is possible to represent φ_1 and φ_2 in terms of analytic functions of a complex variable. The general case where n are positive or negative integers is considered, and analogous results are indicated.

Stefan Bergman, USA

199. J. J. Dronkers, An iteration process for the solution of a boundary problem of a linear partial differential equation of the second order, I (in Dutch), Indagationes Math. no. 2, 11, 103–111 (1949).

For a linear partial second-order differential equation,

$$z_{xx} + az_{yy} + bz_{xy} + cz_x + dz_y + ez + f = 0 \qquad (1)$$

with $z_x(0,y)$ and $z(0,y)$ given, a step-by-step method is proposed. The assumptions are: b_x and c_x, as well as a, d, e and f are continuous with respect to x and the initial conditions analytic in y. If the differential equation is elliptic, the curve $x = 0$ must be closed and only $z(0,y)$ can be prescribed (boundary-value problem).

In order to apply this method the original differential equation is replaced by the system

$$z_x = \alpha u_y + \beta_1 z_y + \gamma_1 z + \delta,\ u_x = \beta_2 z_y + \gamma_2 z \qquad (2)$$

Elimination of u must lead to an equation in z equivalent to the original one. Hence arise nonlinear differential equations for α, β_1, γ_1, ... In certain special cases the determination of the coefficients may become rather simple. Picard's method is then applied to the system (2). The reviewer believes that the method seems to offer difficulties, especially for elliptic boundary-value problems.

Manfred Schaefer, USA

200. N. S. Piskunov, On the characteristic problem for an equation of ultrahyperbolic type (in Russian), Doklady Akad. Nauk SSSR 59, 439–442 (Jan. 1948).

The ultrahyperbolic equation $u_{xx} + u_{yy} - u_{zz} - u_{tt} = 0$ is considered. The values of u are assumed to be given on the characteristic cone $R - r = 0$, where

$$R^2 = x^2 + y^2, r^2 = z^2 + t^2.$$

Under the assumption that the boundary values admit certain developments in Fourier and power series, a solution is constructed for $R \leq r$. Sufficient conditions for uniqueness of the solution are stated.

Courtesy of Mathematical Reviews P. A. Lagerstrom

201. G. L. Polisar, Combination of calculating devices as models with nonsubstituted objects as a method of investigation of dynamic regimes of complex systems (in Russian), Doklady Akad. Nauk SSSR 61, no. 4, 641–644 (Aug. 1948). **Analysis of nonlinear systems of autoregulation by the method of combination of the investigated objects with the electrointegrator** (in Russian), Izv. Akad. SSSR Ser. tekh. Nauk no. 3, 384–395 (Mar. 1949).

The second paper elaborates on the first one. The use of electrical models is discussed for solving problems involving the dynamics of servocontrolled systems. The author mentions (without describing it) an electronic "impulsive" integrator by means of which approximate solutions of 8th-order nonlinear differential equations and partial differential equations may be realized. Evidently this integrator is described in the author's paper in Sudostroenie, no. 1 (1948).

If such integrators are substituted for the controlled systems and combined with actual physical controllers, the order of the system solved is increased by the order of the equations characterizing the added system. There are mentioned applications to ship steering and binary vapor-generating plant.

There is described in some detail the solution of the problem of aircraft-engine liquid-coolant temperature control. The coolant temperature must remain between specified limits determined by engine stall on the low side and fuel detonation on the high side. In this interval the regulator must adjust the rate of liquid circulation so as to ensure minimum drag. By combining the electronic impulse integrator with an actual regulator, the performance of the entire system is determined with a minimum of idealization. Regions of hunting in the system were observed with and without feedback. The author states that excellent agreement was obtained with results from a test on the prototype system.

Walter W. Soroka, USA

Mechanics (Dynamics, Statics, Kinematics)

(See also Revs. 217, 218, 292)

202. Francisco Marfull Sansvisens, Contribution to the study of the stationary motion of inextensible cords (in Spanish), Collectanea Math. 1, 63–108 (1948).

The motion of an inextensible cord, with respect to a set of fixed or moving axes of reference, is said to be stationary if the cord always lies on a curve which is fixed with respect to the axes. In the first chapter of this paper the differential equations of motion of a cord are given in several forms, which are then specialized into the forms that apply in the case of stationary motion. The first part of the second chapter deals with stationary motions in the case in which two of the axes rotate about the third. It is found that only a small number of types of stationary motion are possible; and for each of these the dependence of the rotation of the axes upon the time, and properties of the curve upon which the cord lies, are determined. The latter parts of the chapter deal in the same manner with the subcases in which the rotation of the axes is uniform and uniformly accelerated, respectively. The third chapter deals with the case in which the axes of reference have a rectilinear motion, which may be uniform or not. The greater part of this chapter is devoted to discussions of two applications of the general theory. The first of these is concerned with the form of a submarine cable which is being payed out by a ship moving with constant velocity. The second application is concerned with the form assumed by the anchor chain of a ship moored in a stream. The cases in which the anchor chain is uniform and of constant resistance are both considered.

Courtesy of Mathematical Reviews L. A. MacColl, USA

203. G. Henriot, A treatise on the theory and practice of gears. Vol. 1 (Traité théorique et pratique des engrenages), Dunod, Paris, 1949, 351 pp. Cloth, 6.5 × 9.5 in., 306 figs., approx. $6.75 (1950 fr).

This work contains a summary of the methods and results which have been published only in periodical literature and manufacturers' pamphlets. Its principal source material are the papers of the author and J. Capelle published by the Société d'Études de l'Industrie de l'Engrenage, the British Standard Specifications, and H. E. Merritt's book, *Gears*.

The treatment is more geometric and algebraic than in previous texts even though it includes a thorough reduction to practice of the mathematical results. It is profusely illustrated with line diagrams, photographs, graphs and tabulated data. Numerical examples are given for most of the geometric and algebraic developments.

The opening chapter supplies the reader with the mathematical concepts to be used throughout the book. It discusses vectors and the composition of forces, moments, motions and velocities. It is followed by a general description of the various types of gears including spur and bevel gears, involute and cycloidal gears, helicoidal gears and racks, and a discussion of conjugate surfaces, roulettes and envelopes. More complete discussions follow in separate chapters. The chapters devoted to spur and bevel gears tabulate the definitions of the various terms used along with diagrams. Special methods of generation include the Gleason and Klingelnberg methods. The skew helical gears are then discussed with numerical examples of the design based on given angles between the axes, gear ratio and approximate pressure angle. This is followed by the worm-and-wheel combinations with various shapes of threads and various methods of turning and grinding. There is only a short discussion of the global worm-and-wheel which is known in America as the Hindley worm or Cone worm.

The following chapters discuss, for various types of gears, the detailed tooth action, the arcs of contact, Tredgold traces, interferences, methods of modifying the gears to remove the interferences, the determination of sliding action, the variation of tooth action with variation of center-to-center distance, wear and other operational modifications of the teeth as well as deliberately adjusted tooth shapes. Among the latter, the tooth corrections of Sykes, Gleason and Reinecker are presented.

The stresses and thrusts produced by gear action are derived. These are applied to the determination of the strength and resistance-to-rupture of gear teeth. The Lewis and Merritt formulas are derived and numerical examples of their use are given. The stress factors due to concentration of stress by fillets and changes of section are given. The modification of these stresses by polishing, shot-peening and change of speed is discussed.

Surface pressure and the type of contact are derived by an examination of the mean curvature of the various conjugate sur-

faces. The causes and effects of abrasion are presented, their diagnosis and treatment by lubrication and modification of the teeth are given. The frictional forces and the resultant loss in efficiency are calculated.

The forces on the gear shafts and the stresses (in shear, torsion and bending) are derived. The design of bearings, gear mounts hubs and wheels are discussed. Special new tooth corrections like crowning, trimming and cropping of the summits are presented.

The final chapter deals with the design and calculation of gear trains including epicyclic trains, bevel gears and differentials. Methods are given of approximating desired gear ratios with a small number of gears and practical pitches.

Michael Goldberg, USA

204. N. V. Eremeev, On the premises of the Roberts-Chebyshev theorem (in Russian), Prikl. Mat. Mekh. 13, 115–116 (1949).

The Roberts-Chebyshev theorem states a sufficient condition for the generation of the same trajectory by three different four-bar linkages. The author shows that this condition is not a necessary one, and proves that the necessary and sufficient condition is: the point generating the common trajectory and the three instantaneous centers of rotation of the three connecting rods should be collinear at all times. The condition remains valid when some of the pivots are replaced by crossheads.

A. W. Wundheiler, USA

205. N. I. Levitskiĭ, Equations of constraints for some two-parameter linkages (in Russian), Izv. Akad. Nauk SSSR Ser. Tekh. Nauk no. 2, 174–180 (1949).

The rigid-bar configurations considered are: (a) $OBMM_1$ with $<MBM_1$ constant, O fixed; (b) BMM_1 with $< MBM_1$ constant, B sliding along a straight line; (c) MNN_1M_1, with the angles MNN_1, NN_1M_1 right and NN_1 sliding through a fixed point. The coordinates of M_1 are expressed in terms of those of M and appropriate position parameters of the configuration: an exercise in analytic geometry.

A. W. Wundheiler, USA

206. L. V. Petrokas, Analytical design of cam mechanisms of automatic machines (in Russian), Vest. Inzh. Tekhn. SSSR 184–189 (1948).

The steps of a semigraphical cam-profile determination are described for three simple special cases (one plane, two spatial) in which the follower is a crank whose motion is prescribed. References to the Russian sources of the formulas employed are given.

A. W. Wundheiler, USA

207. Robert Schnurmann, The forces of static and kinetic friction (in English), An. Acad. Brasil. Cienc. no. 2, 20, 197–210 (1948).

According to the author the force of friction between two elements is partly of an electrostatic nature. When the boundary layer has dielectric properties, cycles of slow charging and rapid discharging should account for the well-known "stick-slip" process at low relative velocities. The current explanation by localized melting and welding is disputed.

J. A. Haringx, Holland

208. N. A. Saharnikov, On Frommer's conditions for the existence of a center (in Russian), Prikl. Mat. Mekh. 12, 669–670 (Sept.-Oct. 1948).

Frommer [Math. Ann. 109, 395–424 (1934)] gave three sets of conditions on the coefficients of the differential equation

$$y' = -[x + ax^2 + (2b + \alpha)xy + cy^2]/[y + bx^2 + (2c + \beta)xy + dy^2]$$

under which the point $x = y = 0$ is a center. It was shown by Bautin [Doklady Akad. Nauk URSS (N.S.) 24, 669, 762 (1939)] that some of these criteria were erroneous. The author now gives necessary and sufficient conditions under which the point $x = y = 0$ is a center. These conditions are expressed in terms of algebraic relations connecting the coefficients.

R. Bellman, USA

209. N. N. Bautin, On the behavior of dynamical systems with small violations of the condition of stability of Routh-Hurwitz (in Russian), Prikl. Mat. Mekh. 12, 613–632 (Sept.-Oct. 1948).

Consider a real system $dx_i/dt = \Sigma a_{ij}x_j + X_i(x_1, \ldots, x_n)$, $i = 1, \ldots, n$, where the a_{ij} are constant and X_i is a power series holomorphic around the origin and beginning with terms of degree at least two. It is known that if the characteristic roots λ_i of $A = ||a_{ij}||$ have their real parts negative, the origin is a stable singular point. Liapounoff in his memoir on these questions [Ann. Fac. Sci. Univ. Toulouse 9, 203–474 (1907), reprinted as Ann. of Math. Studies no. 17, Princeton (1947)] has also treated the exceptional cases where: (a) one of the λ_i is zero; (b) two of them are conjugate and pure complex. The author returns to (b) which he discusses rather fully for $n = 2$ when the coefficients depend upon a certain parameter μ. He is particularly interested in the following type of question: Suppose that for $\mu = 0$ there is a bifurcation and the system passes from a stable singular point at the origin, say a stable focus, to an unstable point; is the instability thus arising dangerous or not? For instance, when in the phase plane the focus becomes unstable but there arises a very small stable limit-cycle, it is a consequence of Poincaré's general theory on these questions that the system will continue to remain not far from the origin, for μ sufficiently small, and so the new instability will not be very dangerous. The author applies some of his results to the stability of an airplane in vertical symmetric flight with constant angle of attack.

S. Lefschetz, USA

210. I. S. Gradshteĭn, Differential equations with the derivatives multiplied by a small factor, and Liapounoff's stability theory (in Russian), Doklady Akad. Nauk SSSR 65, 789–792 (1949); Uspekhi Mat. Nauk 4, no. 4 (32), 159–161 (1949).

The papers extend the results of another note of the same author [same source, 64, 441–443 (1949)] and of papers by Tikhonov [Mat. Sbornik 22, p. 64 (1948)] and Friedrichs and Wasow [Duke Math. J. 1946, p. 367]. It proves that the solutions of $\eta\dot{X}_i = f_i(X_k,t)$, with common initial conditions $X_i(t_0,\eta) = a_i$ tend toward the solution $x_i^{(0)}(t)$ of $0 = f_i(X_k,t)$ when $\eta \to 0$, if only the solution of $\dot{x}_i = f_i(x_k,t_0)$, passing through the same point a_i, is asymptotic to $x_i^{(0)}(t_0)$, and under certain regularity assumptions.

A. W. Wundheiler, USA

211. I. M. Volk, On the stability of periodic motions when the equations and their periodic solutions are known only approximately (in Russian), Prikl. Mat. Mekh. 12, 647–650 (Sept.-Oct. 1948).

Consider a system of ordinary first-order differential equations whose right members are meromorphic functions of a small parameter μ. If all but the leading terms with respect to μ are neglected, a simplified system

$$dx_\nu/dt = \mu^{k_\nu}X_\nu(x_1, \ldots x_n; t), \ (\nu = 1,2, \ldots, n)$$

is obtained in which the k_ν may be positive integers. In three previous papers [same source, 10, 559–574 (1946); 11, 433–444

(1947); 12, 29–38 (1948)] the author investigated the existence, for small μ, of periodic solutions x_ν^* of the full system near a given periodic solution x_ν^0 of the simplified system. In the present paper it is shown that for sufficiently small $|\mu|$ the stability character in the sense of Liapounoff is the same for the x_ν^* as for the x_ν^0.
Courtesy of Mathematical Reviews W. Wasow, USA

212. S. V. Kalinin, On the stability of periodic motions in the case when one root is equal to zero (in Russian), Prikl. Mat. Mekh. 12, 671–672 (Sept.-Oct. 1948).

Consider a real system of the general Liapounoff type,

$$dx/dt = X,\ dx_i/dt = \Sigma p_{si}x_i + X_s,$$

$$s = 1, 2, \ldots, n,$$

where X, X_i are holomorphic power series in x and the x_i in $\Sigma x_s^2 < A$, $t > t_0$, and begin with terms of degree not less than 2 in x and the x_i. Moreover, the coefficients of the series are bounded periodic functions of t with period ω. Let c_{sr} be the average of p_{sr} over a period and suppose that: (a) $p_{sr} = c_{sr} + \epsilon f_{sr}(t)$; (b) the characteristic roots of $||c_{sr}||$ all have negative real parts;

$$\text{(c)}\ X = (g + \epsilon G)x^2 + P_x + Q + R,\ P = \Sigma(a_s + 2\epsilon A_s)x_s,$$
$$Q = \Sigma(b_{sr} + \epsilon B_{sr})x_r x_s,$$

where g, a_s and b_{sr} are constant, G, A_s, B_{sr} have zero averages over a period, and R begins with terms of degree at least three in $x_1, \ldots, x_n$. Using Liapounoff's so-called second method the author shows that for ϵ sufficiently small the undisturbed motion is stable when $g > 0$ and unstable when $g < 0$. See Liapounoff's memoir, Ann. Fac. Sci. Univ. Toulouse 9, 203–474 (1907), reprinted as Ann. of Math. Studies no. 17, Princeton (1947).
S. Lefschetz, USA

Gyroscopics, Governors, Servos

(See also Revs. 201, 287, 348)

213. K. Klotter, Progress in the regulation and vibration technique (in German), Z. Ver. dtsch. Ing. 91, 37–44 (Jan. 1949).

A meeting of the sections of the VDI concerned with vibrations and with acoustics took place in Stuttgart in May 1948. The present paper is a report upon the contributions to the vibration field written by the chairman of the meeting. It abstracts nine papers on stability criteria, determination of optimum parameters, use of electrical models, etc; six on the technique of measurements (two on vibration recorders); and five on specific vibration problems (resonance amplitudes, damping in flexure vibrations, etc.). The contributions do not seem of more than average interest. P. Le Corbeiller, USA

214. V. M. Starzhinskiĭ, On auto-oscillations of an electric governor (in Russian), Prikl. Mat. Mekh. 13, 41–50 (1949).

The physical problem is reduced to the mathematical one of determining the possible periodic solutions of the system:

$$J_1\ddot{x} + M_1(\text{sign } x) - k\phi z + M = 0,\ J_2\ddot{y} + M_2(\text{sign } y) - M = 0,$$
$$l\dot{z} + Rz + c_1\dot{x} - c_2(a - y) = 0,$$

where x, y, z are the time-dependent variables and the other letters represent various physical constants. The necessary and sufficient conditions for the existence of periodic solutions (which are stated without proof), lead to a system of algebraic equations connecting the above constants. These equations are solved numerically for different values of the parameters and a table of values is exhibited. R. Bellman, USA

Vibrations, Balancing

(See also Revs. 213, 214, 267)

215. Fritz Sauter, Remarks about vibrations of thin elastic plates (in German), Z. Naturforschung 3a, 548–552 (1948).

Basing his work on solutions previously obtained by Rayleigh [Proc. London Math. Soc. (1) 20, 225–234 (1889)] and H. Lamb [Proc. Roy. Soc. London. Ser. A, 93, 114–128 (1917)], the author presents a general discussion of the shear, extensional and flexural wave motions encountered in bars and plates. The author's purpose is to call these results of Rayleigh and Lamb to the attention of present-day workers, since it appears they have been given insufficient attention in modern literature on the subject. The solutions in question are obtained by the methods of the theory of elasticity, and hold for unidirectional wave propagation by plane strain. They permit discussion of all cases ranging from "short" to "long" wave lengths (as compared with the plate thickness). The "long" waves correspond to the configurations treated in the conventional, thin-plate theory. The general problem is reviewed and the Rayleigh and Lamb results are interpreted. Also, attention is given to the physical reasons why the bending waves in the elementary theory must satisfy a fourth-order differential equation, while the shear and extension waves follow a second-order mechanism. M. Goland, USA

216. Aurel Wintner, A criterion of oscillatory stability, Quart. appl. Math. 7, 115–117 (Apr. 1949).

A sufficient condition is given for the real solutions of

$$x'' + f(t)x = 0 \qquad (1)$$

to be oscillatory. It is proved that if the indefinite integral of $f(t)$ goes to infinity with t, $x(t)$ must be oscillatory, whether $f(t)$ remains positive or not. It is shown moreover that (1) is oscillatory if every solution $x(t) \to 0$ as $t \to \infty$; if the number of zeros of $x(t)$ which lie below t increases faster than t; and if $x(t)/t^{1/2}$ remains bounded for every solution $x(t)$. A corollary of the first theorem is that if $f(t)$ is a positive constant, plus any number (finite or infinite) of simple, periodic terms (harmonic or not), then (1) is oscillatory, a valuable conclusion in engineering problems.
P. Le Corbeiller, USA

217. N. A. Zheleztsov, The method of point transformations and the problem of forced vibrations with "combined" friction (in Russian), Prikl. Mat. Mekh. 13, 3–40 (1949).

"Combined" friction φ is defined (in appropriate units) as follows: (1) for nonzero velocity y the friction φ is a decreasing function $f(y)$ of y only (viscous friction); (2) for $y = 0$ the friction φ depends on the position x and the impressed force $g(t)$ (dry friction):

(a) $\varphi = f(+0)$ for $x - g(t) \leq f(+0)$;

(b) $\varphi = x - g(t)$ for $f(+0) \leq x - g(t) \leq f(-0)$;

(c) $\varphi = f(-0)$ for $f(-0) \leq x - g(t)$.

For g periodic, bounded and piecewise continuous, the author proves that: (1) if $g_{\max} - g_{\min} \leq f(-0) - f(+0)$, there exists an interval of rest positions from $g_{\min} + f(-0)$ to $g_{\max} + f(+0)$, and every motion tends to a rest position for $t \to \infty$; (2) if $g_{\max} - g_{\min} > f(-0) - f(+0)$ (and, at least on bounded motions exists), there exists a motion M with a period T equal to that of g, and every other motion tends toward this motion M; if the viscous friction $f(y)$ is not bounded, the existence of bounded motions is certain.

Special cases of "symmetric" oscillators, and an "antisym-

metric" impressed force are considered. A detailed description of the trajectory distribution in the phase space is given.

A. W. Wundheiler, USA

218. **Nicolas Minorsky, On Van der Pol's oscillator** (in French), C. R. Acad. Sci. Paris 228, no. 1, 60–61 (Jan. 1949).

In this short note on Van der Pol's equation,

$$\ddot{x} - \epsilon(1 - x^2)\,\dot{x} + x = 0,$$

polar coordinates r and θ are introduced in the $(\dot{x}, x)$-plane, $\rho = r^2$ is the energy of the oscillator. Application of the perturbation theory gives for small ϵ a series for $\rho\,(\theta)$ in ascending powers of ϵ. For large values of ϵ the change of energy is studied in the neighborhood of $\epsilon = 0$. W. L. Esmeijer, Holland

219. **J. Beránek, On some problems in the theory of the Lagrangian string and of the Routhian membrane** (in Czech), Publ. Fac. Sci. Univ. Masaryk (Sp. Prir. Fak. Masaryk Univ.) no. 293, 60 pp. (1947).

The author determines the frequencies and frequency densities for (1) the Lagrangian string subject to a sinusoidal force applied at an end particle for a finite or an infinite time, and for a prescribed motion of the free end in a given time interval; (2) for a Routhian membrane acted upon by one or two successively acting impulses at one edge particle. Jaroslav Nekolný, USA

220. **G. F. Carrier, A note on the vibrating string,** Quart. appl. Math. 7, 97–101 (Apr. 1949).

This paper deals with the behavior of an elastic string during oscillations of moderately large amplitudes, and presents a new derivation of the same results which, in terms of elliptic functions, were given by the author in a previous publication. An analogy is pointed out between this problem and the one of boundary layer in fluid dynamics, and criteria of motion stability are discussed for low frequency oscillations of the string. M. Hetényi, USA

Wave Motion, Impact

(See also Revs. 200, 215, 291, 297, 307, 360, 370)

221. **M. M. Fridman, The diffraction of a plane elastic wave by a semi-infinite stress-free rectilinear slit** (in Russian), Doklady Akad. Nauk SSSR 66, 21–24 (1949).

In a previous paper [same source, 60, 1145–1148 (1948); see Rev. 1310, Sept. 1948] the author considered the same problem for a rigid slit. The method is extended to the title problem, and an explicit solution obtained.

Courtesy of Mathematical Reviews V. A. Kostitzin, France

Elasticity Theory

(See also Revs. 221, 231, 238, 240, 251, 261)

222. **Gaston Pigeaud, Strength of materials and elasticity** (Résistance des matériaux et elasticité), Paris, Gauthier-Villars, 1948, 510 pp. Paper, 10 × 6.5 in., 135 figs.

The book contains 19 chapters which may be divided into three groups.

In the first two chapters (56 pages) the author reviews some basic vector theory, mass geometry and numerical integration. Chapters III to VIII (244 pages) form the second group. Chapter III is devoted to experiments on elastic bodies, failure tests, etc. Chapters IV to VI include the general formulation of stress, strain, theories of failure, and equilibrium of elastic bodies. The discussion is limited to linear elasticity. The general character of the three-dimensional solution is indicated. The treatment includes the three harmonic-function solutions. Chapter VI is devoted to plane-strain and plane-stress problems and, in particular, to a study of the isostatic lines. Chapter VIII is devoted to the problem of Saint Venant and the problem of Boussinesq. The topics treated are: deformation of a cylindrical body under uniform compression acting on its lateral surfaces; equilibrium of a hollow circular cylindrical body; equilibrium of a hollow spherical body; and the problem of Saint Venant which consists of equilibrium of a long prismatic body whose two ends are submitted to external forces. The first and second problems of Boussinesq are treated in detail. Thus the equations of elasticity are solved for infinite and semi-infinite boundaries subjected to external forces on a finite part of their boundaries. The Hertz problem of contact pressures between two bodies is presented and exemplified.

From Chapters IX to XIX (239 pages) the material may be described as Strength of Materials. These chapters contain, after a presentation of general principles, the technique for handling beam problems, bending, shear, torsion of bars of arbitrary shape and sections. Formulations are made with the usual approximations used in Strength of Materials. Deflection of beams is treated. The principle of conservation of energy and Castigliano's principle are applied to the study of deflection of beams. The last chapter (62 pages) is devoted to the buckling of bars.

The book is written carefully and clearly. It is valuable to both engineers and research workers. Its level may be considered to be between *Theory of Elasticity* by S. Timoshenko and that of A. E. H. Love. The notation used is due to Lamé.

Large-deformation theory and elastic-wave propagation are untouched. Theorems concerning total potential of an elastic body and the methods which led to approximate solutions are not treated at satisfactory length, in the opinion of this reviewer.

A. Cemal Eringen, USA

223. **J. Walczak, Tangential tensions in oblique sections** (in Polish), Arch. Mech. Stos. 1, 191–202 (1949).

Usually the shear stress on an arbitrary element is expressed as a difference between the total and normal tensions. The author expresses it in terms of the six components of the stress tensor, and discusses simplifications in special cases.

W. Olszak, Poland

224. **G. N. Polozhiĭ, A new method for the solution of some mixed problems of the plane theory of elasticity** (in Russian), Doklady Akad. Nauk SSSR 66, 177–180 (1949).

The author deduces a pair of functional equations for the determination of two analytic functions $\varphi(z)$ and $\psi(z)$ of a complex variable z, giving the solution of the mixed boundary-value problem of two-dimensional isotropic elasticity for a closed simply connected region R bounded by a contour C having a finite number of angular points. On the boundary C the component $\nu(s)$ (s being the arc parameter of C) of the displacement vector in the direction of the normal to C and the tangential component $T(s)$ of the applied force are specified. This mixed boundary value problem was first solved by N.I. Muschelishvili [same source 3, p. 141 (1934)] for regions that can be mapped conformally on a unit circle $|z| \leq 1$ by rational functions. If the region R is bounded by a rectilinear polygon, the equations for the determination of φ and ψ simplify to read:

$$\text{Im}\,[(k+1)\,\varphi'(z)] = -2\mu(d\nu/ds) + T(s)$$

$$\text{Im}\,[\psi'(z)e^{i2\alpha}] = T(s) - \text{Im}\,[\bar{z}\,\varphi''(z)e^{i2\alpha}], \text{ on } C,$$

where k and μ are the elastic constants of the medium, α is the angle formed by the exterior normal to C with the x-axis, and the symbol Im denotes the imaginary part of the expression following

it. The functions $\varphi(z)$ and $\psi(z)$ can be determined with the aid of the formula of Schwarz and by utilizing the continuity properties of stresses and displacements. As an illustration the author solves this mixed boundary value problem for the rectangular region. This solution is new. I. S. Sokolnikoff, USA

225. Henri Pailloux, On the Rayleigh-Ritz method for deformation systems (in French), C. R. Acad. Sci. Paris 226, 1882–1884 (1948).

This note deals with some generalities concerning the use of Lagrangian coordinates in the theory of small free vibrations of an elastic system about a configuration of equilibrium, and in the theory of forced vibrations of such systems. Emphasis is placed upon the relations between the problems concerning free vibrations and problems concerning extrema of quadratic forms in infinitely many variables. In the opinion of the reviewer most of the points made by the author are already rather familiar.
Courtesy of Mathematical Reviews L. A. MacColl, USA

226. Z. Kłebowski and W. Urbanowski, On the unevenness of the stress distribution between the holes of a water-pipe boiler (in Polish), Przegl. Mech. 4–5–6, 118–121 (1949).

Using the results of G. Kirsch (1898), the authors calculate the stress concentration at the edge of a hole in the shell of a water-pipe boiler. The authors show in accordance with Siebel (1929 and 1941) that, in view of plastic strain during the pressing process of a water pipe, it is more reasonable to use the average value of the stress between the holes. J. Nowiński, Poland

227. H. L. Cox, The distortion of a flat rectangular plate in its own plane, Rep. Memo. aero. Res. Counc. Lond. no. 2200, 37 pp. (1945, publ. 1949).

A form of solution for plane-stress problems of the rectangular plate is sought by means of the principle of minimum total (potential) energy. Strain energy and potential of loads are expressed in terms of displacements. The displacements are represented by double Fourier series with the essential addition of certain single series designed to ensure satisfactory differentiability. The variation of the coefficients in these series represents the variation contemplated in the above principle, and the derivatives of the total energy with respect to these coefficients have to vanish. This leads to infinite sets of linear equations in the Fourier coefficients of displacement and of boundary. The work is restricted to normal boundary forces. The general problem of this type is resolved into four subtypes (corresponding in a general way to the resolution of an arbitrary function of x and y in a rectangle, with central origin, into four functions: even-even, even-odd, odd-even, odd-odd, in x and y respectively) representing generalized forms of bending and shearing. Application to cylindrical shells is discussed, in particular the bending of a tube with a cutout. J. N. Goodier, USA

228. D. V. Vaĭnberg, The local stresses in a plane ring-shaped disk under two concentrated forces (in Russian), Prikl. Mat. Mekh. 13, 151–158 (1949).

The paper is concerned with a detailed analysis of local stresses in a plane ring-shaped elastic disk compressed by two concentrated forces applied to the interior and exterior boundaries of the ring. The solution is based on the determination of two analytic functions of a complex variable and follows the methods of solution of plane problems of elasticity developed by N. I. Muschelishvili. As a limiting case, the known solution of the corresponding problem for an infinite strip is obtained.
I. S. Sokolnikoff, USA

229. Eric Reissner, On the theory of thin elastic shells, Reissner Anniv. Vol., pp. 231–247, J. W. Edwards, Ann Arbor, 1949, $6.50.

This paper deals with the theory of rotational symmetric deformations of thin elastic shells of revolution. Equations are first derived for the problem of finite symmetric deflections of shells of revolution. A system of equations are then derived for shallow shells. The basic variables for these last equations are chosen so that the equations apply not only to shells but also to stretching and bending of circular plates. It is shown that the solution of these equations can be expressed in terms of Bessel functions for all paraboloidal shells of constant thickness. It is also shown that solutions can be obtained for certain types of shallow shells of varying thickness. Specific problems are not treated in this paper but certain types of them are mentioned.
R. M. Wingren, USA

Experimental Stress Analysis

(See also Rev. 273)

230. N. I. Prigorovsky and A. K. Preĭss, A study of the state of stress in transparent three-dimensional models by means of beams of parallel polarized light (in Russian), Izv. Akad. Nauk SSSR Ser. tekh. Nauk no. 5, 686–700 (May 1949).

The authors discuss a general photoelastic method for three-dimensional stress analysis. Using thin sections from plastic models into which stresses have been frozen, they combine stress patterns from oblique incidence of parallel light beams, the directions of the principal stresses at each point as determined by the optical effects of converging light (as proposed and used by Hiltscher in 1937), and measurements of lateral deformations obtained after annealing the sections. From the combined data a set of three simple linear equations in σ_1, σ_2, and σ_3 is obtained which leads to the determination of the actual principal stresses and not merely of the maximum shears.

Verification was attempted from grooved shafts in torsion and in bending with excellent agreement between experiment and theory. However, since these special problems can be, and seem to have been, solved by oblique incidence alone it is not clear how they constitute a verification of the method as a whole. The authors have used a new material, NM-44, apparently developed in the Soviet Union, which seems to have the necessary properties for frozen stress patterns.

Unfortunately, the paper is written in a sketchy handbook style. Nonetheless, it represents a significant contribution and will no doubt stimulate considerable activity in three-dimensional photoelasticity. M. M. Frocht, USA

231. Ch. Massonnet, A new apparatus for studying stresses in bent plates and plane pieces (in French), Rev. univ. Min. 5, 41–51 (Feb. 1949).

The apparatus is based on a method of solution for an elastic disk in plane stress under a prescribed distribution p of stress along its contour. The distribution p is replaced by a fictitious distribution P with which stresses are associated as follows: at each point s of the contour the force $P(s)ds$ is treated as a concentrated force applied to the half-plane containing the disk (which is apparently assumed to be convex) bounded by the tangent to the contour at s. The stress at each point in the disk associated with this force is computed from the well-known radial-stress distribution which would occur in the half-plane. Integrating around the contour gives a stress distribution throughout the disk which satisfies equilibrium and compatibility. The problem is to determine the fictitious distribution P so that the associated stress distribution along the contour agrees with the actual pre-

scribed distribution p. The distribution P is obtained by successive approximations beginning with p. The n-th approximant $P^{(n)}$ to P is given by $P^{(n)} = p - B(P^{(n-1)})$ where $B(P^{(k)}) + P^{(k)}$ is the stress distribution along the contour associated with the fictitious distribution $P(k)$.

The author claims that this process will converge if one replaces the function $B(p^{(n-1)})$ in the above by its mean value over the contour. In practice, one stops when $P^{(n)}$ is sufficiently close to $P^{(n-1)}$. For purposes of computation, the integrals are replaced by sums, the contour being broken up into a finite member of segments. The apparatus, devised by the author, performs all the summations mechanically, giving both the stress across a chosen element and the sum of the principal stresses at any given point. The stress distributions, obtained by means of this device are given for some examples, including flexure of a three-member joint and a dam of finite extent under water pressure.

H. J. Greenberg, USA

Rods, Beams, Shafts, Springs, Cables, etc.

(See also Revs. 202, 222, 228, 249, 255)

232. **J. Muttermilch, The accurate curvature formula for eccentrically loaded beams** (in Polish), Arch. Mech. Stos. 1, 203–228 (1949).

The following two problems concerning longitudinally compressed beams are treated: (1) the determination of the largest deflection f for a given load P of eccentricity e; (2) the determination of the slenderness s for a given e and assumed coefficient ψ, which also allows for bending stresses. The two problems can be solved with very good approximation for steel structures of usual dimensions by applying the familiar simplified expression for the curvature of the beam axis. In the case of very slender beams exact calculations are necessary, employing the elliptical function $F(k, \varphi_0)$.

In the second problem the modulus k and the amplitude φ_0 are expressed in terms of e, ψ, the shape of the cross section and the beam material; the calculation of s does not present any difficulties.

But in the first problem, owing to the specific structure of the modulus k and amplitude φ_0, the arrangement of Legendre's tables leads—in the author's opinion—to a dozen or so iterations. To avoid these, there was applied by the author a solution using the finite-difference calculus, which turned out to be also complicated. The author tries therefore an energetic method, assuming the arc of a circle as the axis of a curved rod and obtaining as a result a force P dependent of the angle τ_0 subtended by the arc. Since Euler's value was not obtained for $\tau_0 = 0$, a factor $\pi^2/12$ was applied, resulting in satisfactory approximation for $f/L < 0.25$, L being the natural length of the beam. In the reviewer's opinion this device has destroyed the interpolatory meaning of the energetic solution.

W. Burzyński, Poland

233. **N. K. Arutyunyan, Solution of the problem of the torsion of a rod with a polygonal cross section** (in Russian), Prikl. Mat. Mekh. 13, 107–112 (1949).

An exact solution of the torsion problem for an L-shaped beam is obtained in the form of a doubly-infinite series involving exponential and trigonometric functions. The values of torsional rigidities and shearing stresses are calculated for several L-shaped beams whose sides are of equal lengths but of different widths.

I. S. Sokolnikoff, USA

234. **Ya. S. Uflyand, Torsion of a prismatic bar with a profile limited by arcs of two intersecting circles** (in Russian), Doklady Akad. Nauk SSSR 68, no. 1, 17–20 (1949).

Bipolar coordinates are used to obtain a solution of Saint Venant's torsion problem for a region bounded by two intersecting circular arcs.

I. S. Sokolnikoff, USA

Plates, Disks, Shells, Membranes

(See also Revs. 222, 227, 229, 241, 242, 243, 244, 245, 251)

235. **W. Nowacki, Bending of infinitely long continuous plates** (in Polish), Arch. Mech. Stos. 1, 173–189 (1949).

The author considers a "thin" plate subdivided into longitudinal sections by means of parallel rigid straight-line supports. The uniform section thickness varies from section to section. These loads are symmetric relative to an axis perpendicular to the supports, parallel to them, and constant across the plate. Fourier integrals load to a solution in the form of three-term equations, analogous to the three-moment equations of a continuous beam. The unknowns are the moments at the supports or the slopes at the support. The solutions of Nádái [Bauingenieur, 1921] and Girkmann [*Flächentragwerke*, Vienna, 1948; see Rev. 2, **589**] are special cases of that paper. Other valuable applications are given.

M. T. Huber, Poland

236. **F. Pongratz, The effective width of a reinforced concrete slab loaded with a single load freely supported at two ends** (in German), Öst. Ingen.-Arch. 1, nos. 4-5, 351–370 (1947).

A rectangular reinforced-concrete slab is considered, with two opposite edges free, and the two other edges simply supported. When the load, which is normal to the slab, is uniformly distributed over a small circle, an approximate mathematical solution is obtained by assuming the slab is homogeneous, and by combining the solution for the case when the load is a point load with the solution for the case when the distributed load acts on a circular plate. Numerical results are obtained which are checked by experiment. The case is also considered when the slab is an infinite strip simply supported on the two edges, the slab being acted upon by equal loads equally spaced along the strip.

G. E. Hay, USA

237. **Odone Belluzzi, A simple method to study shells** (in Italian), G. Gen. civ. 86: 217–227, 281–294, 457–461 (1948); 87: 3–17, 115–169 (1949).

This sequence of ten notes concerns simple approximative methods for calculating the stresses in shells and structures composed of shells. Note 1 contains intuitive remarks apparently intended to distinguish between a flexible membrane and a thin shell subject to what are called "membrane" stresses, in which the stress resultants are tangential to the middle surface. Note 2 contains the well-known finite expressions for the stresses and displacements in a shell of revolution subject to a rotationally symmetric membrane state of stress; the results are applied to eight special cases. In note 3 the author proposes to approximate the bending stresses and moments arising at an edge by means of corresponding quantities in a similarly supported sphere tangent to the given shell at the edge. To this end he introduces "elastic coefficients" which he defined in earlier papers [Accad. Sci. Bologna 1945; Il cemento armato, 1945, nos. 4–10; Proc. 13th Congr. of Applied Mech., Schenectady, N. Y., June 1947]; these are displacements and rotations of points on the edge when the distribution of edge loads and moments is uniform and of unit intensity. He gives approximate formulas for these coefficients, and shows that they differ from the exact results computed from the bending theory for three special cases by about one or two per cent. Note 4 contains a discussion of the possible types of support, and the manner in which the elastic coefficients are to be

used to determine the bending stresses to be superposed upon the membrane stresses; the results in each case are applied to a spherical shell subject to its own weight. Note 5 gives a similar procedure for calculating the vertical displacement of the apex. In note 6 the author defines the "rigidity of an edge" of a shell as the magnitude of the uniform moment which must be applied at the edge in order to produce unit rotation, and gives a method for calculating this quantity in terms of his coefficients. In note 7 the author considers structures composed of several shells joined together, possibly with stiffening rings. The joinings are treated as edges, and bending stresses and moments are calculated by means of the "elastic coefficients;" it is supposed that these edges are far enough apart so that the bending stresses and moments arising at one have no sensible effect on the others. Here the author gives 14 examples. In note 8 it is shown that if the displacement in the plane of the edge arising from the supports is negligible, the calculations may be simplified by means of the "rigidity" introduced above. In note 9 the results are applied in detail, with numerical examples, to the "Intze reservoir," which is a circular cylindrical vessel whose bottom consists of a conical shell joined to a spherical cap curved upward. In note 10 there is given a method of tracing the "lines of influence" of the reactions at the supports, and the series closes with a summary of the results and an intuitive explanation of them. The author frequently refers to his recent book, *Scienza delle Costruzioni*, Bologna, Zanichelli, where more details are given.

C. A. Truesdell, USA

238. H. M. Mushtari, A qualitative investigation of the stress state of an elastic shell at small deformations and arbitrary displacements (in Russian), Prikl. Mat. Mekh. 13, 121–134 (1949).

The paper is largely concerned with simplification of the formulation of the theory of elastic shells (based on Hooke's law) and with classification of problems of linear and nonlinear theories of shells. The classification is based on the following conventions: (a) The shell is called thin if its thickness h is of the order of the maximum elongation ϵ permitted by Hooke's law. In this case the quantities of the order h are neglected. (b) The shell is of medium thickness if $h = O(\epsilon^{1/2})$. In this case terms of the order h^2 are neglected. (c) The shell is a membrane if $h = O(\epsilon^2)$. Edge efforts and some problems involving stressed shells of medium thickness are considered. The author's classification contains considerably fewer types than that proposed by W. Z. Chien, Quart. appl. Math. 1, 297–327 (1944); 2, 43–59, 120–135 (1944).

I. S. Sokolnikoff, USA

239. S. E. Birman, On the problem of torsion of a hollow rod with a square section (in Russian), Notes Acad. Sci. USSR (Doklady Akad. Nauk SSSR) 65, no. 4, 461–463 (Apr. 1949).

By superposition of solutions of torsion problems for thin-walled (tubular) rods of square cross section, twisted by forces distributed along the intersections of the lateral faces, deduced by the author in the earlier papers (see Revs. **165, 168**, Feb. 1949), the stress distribution is obtained on the faces of a square rod, twisted by forces applied along a pair of diametrically opposite edges.

I. S. Sokolnikoff, USA

Buckling Problems

(See also Rev. 222)

240. T. M. Charlton, The analysis of elastic stability, Civ. Engng. Lond. 44, 315–317 (June 1949); 385–386 (July 1949).

The author gives an elementary introduction to the application of the energy principle to the analysis of elastic stability, together with a discussion of experimental methods, both leading to well-known results for the buckling of struts. However, the formulation and derivation of the equations are, in the reviewer's opinion, confused and apt to lead to incorrect results, when applied to other buckling problems than those of struts. For a correct formulation and for lucid criticism of derivations like those of the present author, reference may be made to papers by K. Marguerre [Z. angew. Math. Mech. 18, p. 57 (1938) and *Jahrbuch 1938 der Deutschen Versuchsanstalt für Luftfahrt*, p. 252].

Besides, it is stated that Castigliano's principle of least work is not restricted by Hooke's law, but is of universal application to conservative systems. It is well known, however, that this principle must be modified for materials that do not obey Hooke's law in such a way that the strain energy must be replaced by the complementary strain energy ("Ergänzungsarbeit").

W. T. Koiter, Holland

241. E. Kokkinopoulos, Buckling of thin plates (in Greek), Technika Chronika 26, no. 300, 222–226 (June 1949).

This thesis is in two parts. The first part (secs. 1 to 13) deals primarily with the analytical method, and the second one (secs. 14 to 16) with the development and application of a new energy method of determining the critical load and the shape of a buckled thin plate. This new method supersedes all others by being both simple and sufficient, short and clear.

In particular, in sections 11a and 11b, the buckling of a circular plate subjected to peripheral compression with simple and also with built-in edge support is investigated. Absolutely original is a method of determining the shape of the buckled surface. It is found that if the plate is subjected to either a uniform vertical load or a concentrated load, the critical load is the same. Interesting is the case of the plate with built-in edges and supported in the center.

In section 14 the new energy-method formula is derived and the solutions of the four cases, solved analytically in sections 8 and 9, are solved here by the new method. In addition, three cases, presented in paragraphs 5, 6, and 7, and not worked out previously, are dealt with. The comparison of the analytical and the new energy methods clearly shows the expediency of the latter. The accuracy of the solution is very satisfactory. Section 15 investigates the case of a simply supported orthogonal plate subjected to peripheral shearing stress. The solution checks closely with those obtained by S. Timoshenko and F. Hartmann.

The last section 16 deals with the buckling of a simply supported orthogonal plate subjected to vertical bending moments along the plane of the plate. Solutions are obtained by the energy method in the first and second approximation. The results given by these two approximations do not differ much, and they closely check the results obtained by other authors. Plastic buckling is not treated in this dissertation.

Dimitri Kececioglu, USA

242. N. A. Alumyaè, Differential equations of equilibrium of thin elastic shells in the buckling stage (in Russian), Prikl. Mat. Mekh. 13, 95–106 (1949).

The object of this paper is to give a simple derivation of approximate differential equations characterizing the equilibrium of thin elastic shells in the buckled state. The following hypotheses are used in the derivation: (1) the deformations remain small even though the displacements in the buckled state are finite; (2) the distribution of deformations along the thickness of the shell follows the Kirchoff-Love hypothesis; (3) the stressed state of the shell is assumed to be free of moments.

The paper contains a system of differential equations for the determination of critical loads and a qualitative discussion of the form of deformation after the loss of stability. The author

specializes his equations for the case of thin shallow shells and obtains equations identical with those deduced by H. Tsien, Th. von Kármán and L. H. Donnell for flat plates and circular cylindrical shells. Equations deduced by W. Chien in his detailed analysis of thin plates and shells [Quart. appl. Math. 1, 297–327, (1944); 2, 43–59, 120–135 (1944)] also specialize to those given in this paper. I. S. Sokolnikoff, USA

243. **F. Müller-Magyari, Critical stresses of thin-walled platework under axial compression. Part 2** (in German), Öst. Ingen.-Arch. 3, no. 2, 180–196 (1949).

A theoretical solution is given for the buckling of rectangular plates with simply-supported loaded edges under axial compression. The unloaded edges are free to have coplanar displacements; furthermore, they may be elastically restrained against rotation due to either bending or twisting, and against transverse deflections. A stability criterion is derived for the special case where the coplanar displacements are of higher order than the transverse displacements. The critical stresses are then computed by means of a partial-fraction development. Extension to stiffened plates, continuous plates, and plates with edge members is indicated.

Detailed computation is given for an angle section with one leg elastically restrained. A procedure of successive approximation is also given for determining the crippling stress of open sections such as the Z-sections, the hat sections, etc.

There are some misprints on pages 190 and 192.

Conrad C. Wan, USA

244. **D. M. A. Leggett and H. G. Hopkins, Sandwich panels and cylinders under compressive end loads,** Rep. Memo. aero. Res. Counc. Lond. no. 2262, 22 pp. (1942, publ. 1949).

This pioneer work on flat and curved sandwich construction employs an approximate analysis based on strain-energy considerations. It examines the buckling under end load of flat and slightly curved sandwich panels with edges simply supported, and the behavior under end load and flexure of a complete cylinder. Attention is confined to sandwich employing dural faces and balsa cores, which was found in an earlier study (same source, no. 1987) to make the most efficient struts of several types of sandwich investigated. The authors determine the weight of this type of construction and state that for wings with rib spacing not more than one-third the distance between spars, sandwich is more efficient than sheet-stringer construction by an amount which depends on, and increases with, the curvature.

In the opinion of the reviewer, changes that have taken place since 1942 limit the present usefulness of the report in a number of respects: (a) new types of core material are available which were not considered in the study which led to the selection of balsa as the most efficient; (b) the restriction of the rib spacing to one-third the distance between spars is not representative of current sandwich practice; and (c) the efficiency study on which the comparison with sheet-stringer construction was evidently based, restricts consideration to relatively low load per chordwise inch.

S. B. Batdorf, USA

245. **Giulio Krall and Domenico Caligo, On the buckling load of a vault** (in Italian), Pubbl. Inst. appl. Calcolo no. 219, 22 pp. (1948); reprint from R. C. Accad. Lincei ser. 8, vol. 1, no. 12, 1281–1295 and 15–21 (1948).

This paper presents a method of calculating the buckling loads of cylindrical vaults, such as are often built in Europe of reinforced concrete. The vault considered is a curved panel, supported either simply or rigidly along the curved edges, and carrying a stringer along the straight edges. The cross section of the vault is assumed to be either circular or cycloidal, but most of the basic development holds for any arbitrary section.

The loading consists of the weight of the vault. Only membrane stresses are assumed to exist before buckling; this restricts the types of loading which may be considered.

General expressions for the strains and for the curvatures are derived for any cross-sectional shape taking into account second-order quantities; the strain energy and the external work are then calculated with the aid of these expresssions for the sections considered. The deflections are assumed in terms of trigonometric series, the coefficients of whose terms are determined by minimization of the total potential energy. Two types of buckling are considered, one involving no bending and one involving no extension. The former mode holds only for small length to radius ratios.

Part 2 of this note considers vaults with more than one field in the axial direction, and presents the results in tabular form. The problems arising because of long-time creep in the concrete are briefly mentioned. Bruno A. Boley, USA

Joints and Joining Methods

246. **J. R. Stitt, Effect of temperature gradients upon the introduction of residual stresses in weldments or other structures,** Proc. Soc. Exp. Stress Anal. 5, no. 1, 67–70 (1947).

A calculation of thermal stresses shows that for mild steel of yield point 33,000 psi, a temperature differential of 340 F is required to produce yield-point stress if the geometry of the structure is symmetric, and compression and tension areas opposing each other are balanced, as well as compression and tension stresses. If these conditions are not satisfied, 140 F suffice to produce yield-point stress, and any increase in the temperature differential will produce plastic flow.

These calculations were tested on a specimen consisting of a cylindric shaft with two collars, over which a pipe sleeve was slipped and welded to the collars. During cooling from 1200 F the maximum average temperature differential observed was 172 F; it caused plastic flow in the sleeve, which produced 22,600 psi in compression at the final room temperature. This confirms the calculations, after they are corrected for change of yield point, elastic modulus and thermal-expansion coefficient during the cooling. E. Siebel, Germany

247. **J. Heuschkel, Steel properties related to welded performance,** Weld. Res. Suppl. 14, 135–152 (Mar. 1949).

This paper reports a comparative study of previously reported welded tee-joint bend tests on $^1/_2$-in. thick plate materials. The double-fillet welded tee was on the tension side of the bend, and the performance of the joint measured by the area of the test-load vs. deflection diagram. This area, representative of energy absorbed, is compared to that for low-carbon structural steel as a basis for rating welded performance. These performance ratings plotted against prewelded strengths, hardness, and carbon content provide a means of studying the influence of such factors for the types of steel tested. Henry A. Lepper, Jr., USA

248. **H. G. Geerlings, Welding of alloyed steels,** Ingenieur 's-Gravenhage 61, 1–12 (Jan. 1949).

The author deals with the measures to be taken in welding alloyed steels. For low- and medium-alloyed steels the required welding conditions can be determined by means of Doan's procedure or with aid of S curves. The way in which these curves, which apply to isothermal transformations, are used for continuous cooling, is indicated. The choice between electrodes of the same composition as the parent material and austenitic

electrodes is treated in the discussion of welding Mo and Cr-Mo steels. Some fatigue tests have been made on welded medium-alloyed steel tubes.

For the welding of stainless steels the author arrives at the following conclusions: Austenitic steel has a good weldability if the quality of parent material and electrodes is suitable for welding; methods are discussed to test the weldability. The weldability of martensitic steels is poor. Ferritic steels are weldable. Thereby greater strength may be obtained by using austenitic instead of ferritic electrodes. Ferritic-martensitic steels need a heat-treatment after welding. J. J. Koch, Holland

249. F. G. Riessauw and D. A. Passelecq, Study of joints in reinforced concrete (in French), Ann. Trav. publics Belg. 101: Feb., 53–79; Apr., 223–268 (1948).

This paper deals with several tests made by the authors at the Reinforced Concrete Laboratory of the University of Gand, directed by Magnel. It gives an interesting theoretical solution of the stress-distribution problem in reinforced-concrete rockers, which agrees well with strain measurements and permits calculation, with good approximation, of the tensile stresses in the middle longitudinal plane of the rockers, through the contact lines of the joint.

Several types of joints were investigated: (a) *Joint with cement-mortar contact strip* (proposed by Freyssinet). This joint has satisfactory behavior if the rotations are small (as in arch hinges whose rotations are about 0.5—1%); relatively large rotations (by pendular supports of girder bridges whose rotations are at least 1.8%) produce remarkable load eccentricities and consequently cracks in the mortar contact strip which in practical applications should be avoided. The velocity of loading does not particularly influence the deformation capacity of the mortar contact strip. (b) *Joint with concrete-to-concrete contact.* This joint-type is to be avoided. (c) *Joint with steel-to-steel contact.* This joint consists of two steel strips, each of which is immersed in the head of one of the connected members and carries one contact surface. It allows large joint rotations, improves the behavior of the concrete placed in the neighborhood of the contact zone, but produces high tensile stresses in the middle longitudinal plane of the connected members. (d) *Joint with lead contact strip.* Owing to the remarkable elastic flow of lead this joint is sometimes regarded with disfavor. On the basis of their test results, the authors affirm that this view is not justified, provided the contact strips are designed to take into account the deformation laws of lead [see O. Veit, Forschungsarb. Geb. Eisenbetons 43 (1935)].

The testing arrangement which employs the prestressed-concrete technique is interesting. Giulio Ceradini, Italy

Structures

(See also Revs. 237, 243, 249, 384)

250. Pei-Ping Chen, The equivalent loading method and the equivalent beam method, Quart. appl. Math. 7, 183–200 (1949).

Three-dimensional prismatic frameworks treated in a previous paper by the author [J. Franklin Inst. 238, 325–334 (1944)] have been extended to include nonuniform cross sections. The equivalent loading method, developed for nonuniform loading, is now supplemented by the equivalent beam method for variable cross sections. These methods amount to a type of symmetrization resulting from the relaxation of the loading conditions and the cross-sectional form. A number of illustrative examples are given. B. R. Seth, USA

251. W. Flügge, Statics of the wing (in French), Off. nat. Étud. Rech. aéro. Rep. no. 24, 52 pp. (1949).

The author develops the equations of the deformations and stresses in a sweptback wing of constant cross section due to extension, uniform bending and uniform torsion. It is shown that especially the elastic coupling between torsion and bending is considerable. Two numerical examples, (for a rectangular box section and for an airfoil section) illustrate the results.

W. T. Koiter, Holland

252. Heinrich Wix and Hans Dornau, A contribution to the method of moment distribution, Bautechnik 26, no. 1, 20–22 (Jan. 1949).

This paper deals with lateral loads on bents with skewed legs or girders. However the material is not new, having been presented in detail in the book by Hardy Cross and N. D. Morgan, *Continuous Frames of Reinforced Concrete*, John Wiley and Sons, New York, 1934. The writers are apparently unfamiliar with the voluminous material on moment distribution that has been published in the United States. N. M. Newmark, USA

253. A. Winokur, Checking of frames computed by the Cross method, (in Polish), Inzyn. Budown. no. 3, 129–133 (1949).

The author shows, in the case of two-storied frames, that results correctly obtained by Cross' method yield a zero sum of horizontal forces (for each story), a zero sum of moments at every joint and invariance of angles between the members meeting at one joint. J. Nowiński, Poland

254. J. Czulak, Calculation of the deformation of a three-hinged arch (in Polish), Inzyn. Budown. no. 4, 206–210 (1949).

The main difficulty in elastic-arch computations is presented by the integrations in Bresse's formulas because of the arc differentials ds which they contain. In three-hinged arches this difficulty is aggravated by the discontinuity of the deformed axis of the arch. The author seeks to overcome this difficulty by using for ds an expression of W. Wierzbicki (Mem. Ass. Int. Ponts Charpentes, 1937/38) based on a least-squares approach. The author gives formulas for the components of the displacement of the center hinge of a three-hinged arch and the corresponding influence lines. The method uses geometric addition of deformation, which the author considers the safest in unconventional cases. K. Zarankiewicz Poland

255. W. Adamski, Reinforced walls (in Polish), Inzyn. Budown. no. 3, 111–118 (1949).

The author gives an account of his own study of some tens of columns and beams of reinforced brick wall. The strength of columns of slenderness ratio 1:9.25, longitudinally reinforced and equipped with stirrups, or transversally reinforced with deployed metal, was about 2.5 of their nonreinforced counterparts. The author concludes that there is a considerable reserve of strength in conventional walls and that, after appropriate reinforcement, inferior materials (half bricks, raw rubble etc.) can be used for the remaining parts of the walls. J. Nowiński, Poland

256. H. F. Michielsen, The stress analysis of arbitrarily shaped wing structures with shear stressed skin and walls, J. aero. Sci. 16, 659–673 (Nov. 1949.)

When the elasticity of the ribs is considered, wing structures become highly indeterminate. On the basis of usually justifiable assumptions, a procedure is developed for the stress analysis of wing structures including the effects of rib distortion. The formulation uses the principle of least work in the ordinary manner and attention is concentrated on cases in which the external

loads are resisted entirely by membrane stresses in the web elements, i.e., skin panels, spar webs, and ribs.

John E. Goldberg, USA

257. **P. Allaart, An investigation of wind pressure on movable bridges with horizontal axis of rotation** (in Dutch), Ingenieur 's-Gravenhage 61, 11B–16B (Feb. 25, 1949).

To design the operating mechanism of bascule bridges it is important to know how the wind pressures vary with slope angle of the bridge-deck. Experiments were performed to this end in the National Laboratory for Aeronautics in Amsterdam. The results are given in graphs from which for a given bridge the moment with respect to the axis of rotation may be computed.

The moment-slope angle relation appears to depend on the ratio of the width to the span of the bridge, on the roughness of the surface of the bridge floor, and on the presence of embankments. In the latter case considerable upward pressures occur with small slope angles and even with closed bridge.

P. P. Bijlaard, USA

258. **Ch. Mallet, Prestressed concrete. Some North-African realizations** (in French), Bull. Centre Étud. Constr. Génie civ. Hyd. Fluviale 3, 69–127 (1948).

A general discussion precedes a description of some dams and bridges of prestressed concrete. Ed.

Rheology (Plastic, Viscoplastic Flow)

(See also Revs. 246, 272)

259. **P. M. Ogibalov, On the duration of the viscoplastic flow of a substance after an impulsive load** (in Russian), Uchenîe Zapiski Moskov. Gos. Univ. Mekhanika 122 (II), 183–192 (1948).

A plastic material is considered which deforms in shear under the stress $\tau_{\max} = \pm k + \mu v + h \tan \gamma$, the three terms being respectively the initial quasi-static yield stress, a viscous rate of strain term, and a linear, work-hardening term; elastic strains being neglected. A circular cylinder of rigid material is embedded in a plane slab of the plastic material, with its axis normal to the slab. Initially the cylinder is given an impulsive angular velocity ω_0, and subsequently moves under the influence only of the torque imposed by the plastic material. The inertia of the plastic material is neglected in comparison with that of the cylinder: this gives the stress distribution immediately, for $\tau_{r\theta}$ is the only nonvanishing stress component. Without this simplification the problem would be one of plastic wave propagation. A third-order partial differential equation is obtained for the angular displacement in the plastic material, a solution of which is expressed in terms of three arbitrary functions. The boundary conditions determine a second-order ordinary nonlinear differential equation, which is solved for the case of no work hardening. The extent of the region straining at any time is determined, and the time to bring the cylinder to rest. E. H. Lee, USA

260. **D. Trifan, A new theory of plastic flow,** Quart. appl. Math. 7, 201–211 (July 1949).

A new stress-strain relation for an isotropic, incompressible strain-hardening material exhibiting a gradual transition from the elastic to the plastic state is given in the form

$$s_{ij}^{*} = 2G_0\epsilon_{ij}^{*} - p(E)E^{*}\epsilon_{ij},$$

where ϵ_{ij} is the strain tensor, s_{ij} the stress deviator, E the strain intensity, and * denotes rate of change with respect to a parameter t. The function $p(E)$ can be determined from a tensile test. It is shown that, for a given state of strain throughout the body and given velocities on the surface such that the surface integral of the normal velocity component vanishes, the rate of change of strain is uniquely determined throughout the body. A minimum principle for loading, together with certain extensions, is given. Through the use of perturbation scheme (odd powers), this theory is compared with a theory of plastic deformation, $s_{ij} = 2G'(E)\epsilon_{ij}$. The linear and third-order terms are found to agree for both theories. Conditions are also given for agreement of higher-order terms.

G. H. Handelman, USA

261. **V. G. Osipov, Relation between elongation and shear strain at different kinds of deformation** and **N. N. Davidenkov, on the paper, "Relation between elongation and shear etc."** (in Russian), Zavodsk. Lab. 15, no. 5, 580–583 (1949).

A formula is derived for the shear strain of a specimen subjected to pure tension. The author arrives at the expression

$$\gamma = \sinh 1.5e_1,$$

where e is the so-called "natural strain" $\log (1 + \epsilon_1)$. He is wrong in his dispute with Davidenkov, because he lost sight of the fact that in Davidenkov's formula $g = 1.5e_1$, g was the "natural" and not the actual shear strain, the relation between these two being of the same nature as between e and ϵ. The question whether the natural strain has a physical meaning or not is of no practical importance. M. Gololobov, Czechoslovakia

262. **Aris Phillips, Variational principles in the theory of finite plastic deformations,** Quart. appl. Math. 7, 110–114 (Apr. 1949).

Some years ago L. M. Kachanov proved two variational theorems for the theory of infinitesimal plastic deformations. In the present paper the author proves the two corresponding variational principles for the theory of finite plastic deformations by using Lagrangian coordinates and taking strain hardening into account.

The proof is based upon the equations of equilibrium, the boundary conditions and the relation between stresses and finite strains. The latter is a generalization of the similar relation between stresses and infinitesimal strains in the theory of infinitesimal plastic deformations.

In the first variational principle, it is proved that the actual value of the work of deformation is a minimum when the displacements and strains are varied. In the second variational principle it is proved that the actual value of the complementary work of deformation is a minimum when the stresses are varied.

The two variational principles for infinitesimal plastic deformation are special cases of the variational principles given in this paper. L. Föppl, Germany

263. **A. I. Gubanov, Immersion of a cylinder in an elastoviscous medium** (in Russian), Zh. tekh. Fiz. 17, 253–262 (1947).

The paper deals with a problem analogous to the one solved by Riemann who studied the elastic half space under the weight of a perfectly rigid cylinder on its surface. The author substitutes for the elastic half space an elastoviscous medium whose differential equation (without body forces) is:

$$\rho\ddot{u} + \rho t\dddot{u} = \lambda\nabla(\nabla u) + t(\lambda + G)\nabla(\nabla\dot{u}) + \eta\nabla^2\dot{u},$$

as suggested in 1945 by Ya. I. Frenkel. Here u is the vector displacement, ρ density, λ and G elastic constants, η viscosity, $t = \eta/G$ the time of relaxation.

The author's solution differs from the one given previously by M. A. Slyozkin [Prikl. Mat. Mekh. 9, 233 (1945)] in that he

takes into account the variation of the velocity of immersion. The author applies the Laplace transform and Mellin integral, neglecting the inertia terms in the general equation. For the value of the resistance of the medium to the immersion of a cylinder at constant velocity he finds

$$F = 8\eta av \{1 - \tfrac{1}{2}\exp(-t/r) - \tfrac{1}{2}\exp[(-t/r)\lambda/(\lambda + 2G)]\}.$$

When the cylinder strikes the surface of the medium with a small velocity v_0 (m is mass, a radius of the cylinder) the author obtains the depth of immersion as $s = (mv_0/8\eta a)[1 - \exp(-8\eta at/m)]$. When the medium becomes a solid body, this solution becomes $s = v_0[m(\lambda + 2G)/8aG(\lambda + G)]^{1/2}\sin t[8aG(\lambda + G)/m(\lambda + 2G)^{1/2}]$.

The paper considers, as a numerical example for experimental checking, the case of colophane at a temperature of above 56.5 C, at which temperature it has the characteristics of an elastoviscous medium with known material constants. Finally the author points to the possibility of generalizing his method by taking advantage of the formulas of the theory of elasticity, substituting in them the constants λ and G by properly determined differential operators, as demonstrated by Frenkel.

M. T. Huber, Poland

Failure, Mechanics of Solids State

(See also Revs. 275, 276, 382, 385)

264. S. B. Joshi and R. N. Joshi, Oblique shear, reprinted from Proc. 27th ann. Session Instn. Engrs. India–Bombay Center, 23 pp. 17 figs. (1949?).

This paper deals with a new "theory" on the shear strength of materials like concrete, masonry and cast iron, which show a great difference in their compressive and tensile strengths. The results are not very convincing. J. A. Haringx, Holland

265. C. F. Tipper, The fracture of metals, Metallurgia 39, no. 231, 133–137 (Jan. 1949).

The author presents a review of current information, particularly about brittle fracture, including some of the results obtained in the study of mild steel, conducted for the Admiralty Ship Welding Sub-Committee. M. B. Millenson, USA

Design Factors, Meaning of Material Tests

(See Revs. 267, 274, 278)

Material Test Techniques

(See also Revs. 248, 249)

266. I. I. Kornilov, Centrifugal method for testing the strength of metals and alloys at high temperatures (in Russian), Zavodsk. Lab. 15, 76–82 (Jan. 1949).

In order to determine the strength and rigidity of various ferrous and nonferrous alloys at temperatures ranging from room temperature to 1200 C, a machine was constructed which applies loads and deflections by means of the action of centrifugal forces. Various alloys were tested and the strength and rigidity compared. At high temperatures these tests can also be used to determine the chemical stability of the alloy. Andrew Brodsky, USA

267. Ai-Ting Yu and Bruce G. Johnston, A method for vibration fatigue tests of stranded conductor, Proc. Soc. exp. Stress Anal. 4, no. 2, 1–6 (1949).

A machine is described for conducting fatigue tests of steel cables or wires. The cable is strung over a free length up to 30 ft at a tension which may be adjusted up to 15,000 lb. It is maintained in lateral vibration with a double amplitude up to 2 in. by a push-pull motor of the moving-coil type. The motor is supplied from a 120-watt amplifier with sinusoidal current at frequencies between 15 and 200 cps. The amplitude of the current is controlled to maintain the vibration amplitude of the cable constant through a feed-back arrangement which is coupled to the cable with a pickup of the variable-reluctance type. This pickup has the advantage of not requiring any attachment to the cable itself. The tension in the cable is measured with a ring dynamometer and the vibration amplitude and frequency are measured with a stroboscope and scale. The machine is cut off automatically when one of the wires breaks and the time to failure, proportional to the number of cycles to failure, is recorded by stopping an electric clock. Walter Ramberg, USA

268. H. E. Van Valkenburg, The theory of ultrasonic materials testing, Mech. Engng. 71, 817–820 (Oct. 1949).

An expository paper.

Mechanical Properties of Specific Materials

(See also Revs. 248, 255, 265)

269. A. L. Johnson, Determination of factors affecting lamination in structural clay products, Iowa State Coll. engng. exp. Sta. Bull. 47, no. 164, 32 pp. (Dec. 29, 1948).

Clay materials normally used in brick manufacture were studied with respect to their gradation, clay mineral type and content, plasticity (yield value times angular deformation), and sulfate and carbonate content. These clays contained either 25 to 50% of kaolinite or 5 to 10% of illite. The illite clay samples contained carbonates in addition to other impurities. It is suggested that slip planes formed in extrusion may be prevented from fusing together at burning temperatures by the development of gases whose outward passage is impeded by impervious glassy layers. It is recommended that indications of this work be checked against plant experience in an attempt to answer the question of the effect of (a) type of clay mineral, (b) presence of carbonate, and (c) plasticity, as a function of the sulfate content, on the durability of structural clay products. Hans F. Winterkorn, USA

270. R. Cabart, L. Guillet and R. Le Roux, The elastic properties of metallic alloys, J. Inst. Metals 16, 391–402 (Feb. 1949).

The authors have investigated the elastic modulus and the internal friction (logarithmic decrement of amplitude) of Cu-Sn and Cu-Zn alloys, applying a dynamic method. The cylindrical test bar (140 × 8 to 12 mm), fixed at its center and vertically placed between two electrodes, was brought in resonance by the periodic electrostatic force of the top electrode, while the bottom electrode acted as a microphone.

The alloys were prepared partly by rolling (α-bronze and α-brass), chill casting (bronzes) and sand casting (brasses), followed by long annealing (100 to 1000 hr) and slow cooling (1 C/min), to obtain maximum soundness and homogeneity. The composition vs. modulus curves show a pronounced maximum in the Cu-Sn-system at 20–22% Sn, referring to what the authors call the δ-phase (which is not stable below approx. 500 C) and in the Cu-Zn-system at 60–68% Zn, referring to the γ-phase. These maxima correspond to minima on the composition vs. logarithmic-decrement curves. This behavior is regarded as typical for the phases with multiatomic cells, which have a less marked metallic

character. The logarithmic-decrement vs. composition curves, which are similar to the electrical conductivity vs. composition curves, possess maxima for β-brass (body-centered cubic), ϵ-brass (hexagonal) and ϵ-bronze (24–26% Sn, hexagonal), in agreement with the pronounced metallic character of these phases.

J. H. Palm, Holland

271. W. Kast, L. Flaschner, and E. Winkler, Orientation and strength of cellulose fibers (in German), Kolloid Z. 111, 1–6 (Oct. 1948).

This paper describes an investigation aimed at correlating the degree of orientation of the structure of cellulose fibers with their mechanical strengths. Various degrees of orientation were produced in two types of fibers, each of which had been subjected to two kinds of handling during production. Mechanical stretching was the direct cause of the orientation in each case. X-ray interference patterns permitted measurement of the degree of orientation, which showed close correlation with the stretching process. For each material there appeared to be a limiting degree of orientation which was not exceeded by increasing the amount of stretching. It is concluded that for a given material the strength is increased by improving the orientation, but that good orientation, although necessary, is not a sufficient condition for high strength which depends on the structure.

M. P. White, USA

272. Pierre Laurent and Michel Eudier, On the influence of plastic deformations on the modulus of elasticity (in French), C. R. Acad. Sci. Paris 228, 225–226 (Jan. 1949).

The authors present the results of two tests in which the modulus of elasticity of an Al-Cu alloy with 9.7% Cu was measured as a function of time for a constant load at room temperature, and then as a function of load. In the first test where the applied load was 18 kg/mm², the modulus of elasticity was found to change from its initial value of 7300 kg/mm² to a value of 6850 kg/mm² at the end of 64 hr. The strain due to creep after 64 hr was 0.209. In the second test the applied load was increased from 18.3 kg/mm² to 21.3 kg/mm² accompanied by a corresponding decrease in the modulus of elasticity from 7300 kg/mm² to 6850 kg/mm². The authors conclude that the mechanism which causes the change in the modulus of elasticity under the above condition of loading is essentially the same as the mechanism for plastic deformation. Paul Lieber, USA

273. L. W. Nickols, Internal stresses in hardened and dimensionally stabilized steel, Engineering 167, no. 4333, 122–123 (Feb. 1949).

Steel gages and end standards must be hardened in order to reduce wear and the effect of accidental damage of the gaging faces. Since hardened steel is not dimensionally stable it must be subjected to a stabilizing process. At the National Physical Laboratory this process consists of heating at 150 C for about 10 hr and then allowing the steel to cool slowly in the furnace. Current opinion to the contrary and according to the author, steel thus stabilized is affected by high internal stresses. This has been shown by testing a hardened and then stabilized cylindrical bar, 1 in. in diam and 10 in. long, of 0.9% C tool steel. The internal stresses were determined by progressively reducing the diameter of the bar and measuring the change of its length. In the external layers compressives stresses with a maximum of about 32,000 psi, and in the internal layers tensile stresses about 40,000 psi were found. The bar, reduced to a diameter of 0.46 in., has proved to be dimensionally stable. The author's interpretation is that dimensional instability of quenched steel is not due to the internal stresses, which are not removed by the stabilizing process, but to some changes in the structure of steel.

Giulio Ceradini, Italy

274. E. P. Klier, F. C. Wagner and M. Gensamer, The correlation of laboratory tests with full scale ship plate fracture tests Weld. Res. Suppl. 13, 71–96 (Feb. 1948).

The authors used standard impact tests on merchant-vessel quality ship plate to establish the energy absorption vs. temperature curves. The values obtained do not correlate with the behavior of the same steels in large plates as determined by other researchers. The purpose of the investigation reported in this paper is to find a test which would allow the prediction of the behavior of steel in actual service. Looking for this test the authors found it convenient to study the influence on energy absorption measurements of several charges introduced in Charpy type specimens, and of strain aging and thickness of the plate of which the specimens were taken from. The authors finally developed a slow bend test which according to their tests correlates, but for a few exceptions, with the large-plate (72-in. width) test results.

A. J. Durelli, USA

275. N. D. Tomashov and V. A. Titov, Corrosion test of wire samples with simultaneous application of constant tension (in Russian), Zavodsk. Lab. 15, 48–53 (Jan. 1949).

Tests were performed on steel wires subjected to tension and simultaneously placed in mine water to cause corrosion. Various amounts of tension were applied and the time required to cause fracture was determined in each case.

Further tests were performed to measure the electric resistance and strength of wires after corrosion and simultaneous tension.

Andrew Brodsky, USA

276. A. J. Gould, Corrosion-fatigue of steel under asymmetric stress in sea water, J. Iron Steel Inst. 161, 11–16 (Jan. 1949).

Results of push-pull type fatigue with a superimposed tensile stress are reported in this paper for mild-steel specimens having polished, scaled, or scaled and pickled surfaces subjected to air and aerated sea water. Although the numerous variables of such a program have not been thoroughly explored, certain facts seem to stand out. Push-pull fatigue specimens require roughly five times the number of loading cycles for failure as for the rotating beam, other factors, such as surface, stress, and corrosive conditions being equal. This curious effect is explained on the basis of a more severe cell action in the case of the rotating beam. The fatigue strength seems to be insensitive to the degree of superimposed tension under conditions of a corrosive atmosphere. When the oxide coating on the surface was broken, fatigue strengths were considerably lower than when the scale was unbroken. This effect is again attributed, at least partially, to the more severe galvanic action. L. F. Coffin, Jr., USA

277. J. M. Hodge, R. D. Manning and H. M. Reichhold, The effect of ferrite grain size on notch toughness, J. Metals 1, 233–241 (Mar. 1949).

This is the first quantitative determination of the effects of grain size and nickel content on the Charpy impact-test transition temperature in which other factors affecting the result were carefully controlled. The results were:

(1) The keyhole-notch Charpy transition temperature of 0.02%-carbon steels is a straight-line function of the ferrite grain size, the transition temperature being raised 30 F by an increase in ferrite grain size of one A.S.T.M. grain-size number.

(2) The keyhole-notch Charpy transition temperature is lowered 60 F at a constant ferrite grain size in 0.02%-carbon steels by alloying with 3.64% nickel.

(3) The room-temperature impact value in the keyhole Charpy impact test for each steel is also a function of the ferrite grain size, increasing as the grain size decreases.

(4) At a given ferrite-grain size, the room-temperature impact value in the keyhole Charpy impact test is unaffected by alloying with 3.64% nickel. Maxwell Gensamer, USA

278. D. Rosenthal, G. Sines, and G. Zizicas, Effect of residual compression on fatigue, Weld. Res. Suppl. 14, 98–103 (Mar. 1949).

Fatigue tests were made on 61S aluminum-alloy notched specimens in a Rayflex fatigue tester. The material was annealed to stress-free condition, one set fatigue-tested in this form, and two sets fatigue-tested after static loading to produce two magnitudes of residual compressive stress. No significant effect of the residual stress was found in the fatigue tests.

Henry A. Lepper, Jr., USA

279. R. M. Seborg, M. A. Millett, and A. J. Stamm, Heat-stabilized compressed wood (Staypak) For. Prod. Lab. Rep. no. 1580, 1–11, 4 tables and 8 figs (Dec. 1948).

Staypak is a new wood material developed by the authors at the Forest Products Laboratory. The process is covered by USA Patent 2,453,579; it consists in manufacturing solid flat sawn nonresinous wood and veneer spread with glue and assembled in parallel laminated or cross-banded form; the material is compressed under such conditions (temperature moisture content, time of heating) that some flow of the lignin occurs, which eliminates the internal stresses resulting from the compression and consequently the spring-back tendency.

Staypak has considerably higher strength properties than those of the normal wood from which it is made; the strength increases about in direct proportion to the compression. Over normal densified wood (densified under conditions which do not cause flow of lignin) Staypak has the advantage of a satisfactory stability: over Compreg (resin-treated compressed wood, which is stabilized by the resin) it has the advantage of an about double impact strength and 25% greater tensile and bending strength; shear strength is however appreciably lower for Staypak than for Compreg. Staypak swells also more than Compreg.

The most suitable manufacturing conditions, the species to be employed and the potential uses of the material are discussed in the paper on the basis of several tests. The physical and strength properties of Staypak are systematically investigated.

Giulio Ceradini, Italy

Mechanics of Forming and Cutting

(See also Rev. 269)

280. Z. Wusatowski, Draft, spread and elongation in the hot rolling process (in Polish), Prace bad. Główn. Inst. Metal. Odlewn. 1, 27–58 (1949).

In this paper l_1, C_1, h_1 and l_2, C_2, h_2 are the lengths, widths and thicknesses of a prismatic steel bar before and after the pass, respectively. D is the roll diameter. The coefficients of elongation $\lambda = l_2/l_1$, of spread $\beta = C_2/C_1$, of draft $\gamma = h_2/h_1$ and the coefficients $\delta = h_1/C_1$, $\epsilon = h_1/D$ are defined with the help of these values. On the basis of many experimental results available and the author's own measurements, the following empirical, multiply exponential formula for the spread was obtained:

$$\beta = \gamma \exp[-10 \exp(-1.269\ \delta.\epsilon \exp 0.556)].$$

The condition $\beta\gamma\lambda = 1$ must always be satisfied.

W. Burzyński, Poland.

Hydraulics; Cavitation; Transport

(See also Revs. 299, 300, 330, 363)

281. Garbis H. Keulegan, Theory of flow through short tubes with smooth and corrugated surfaces and with square edged entrances, Highw. Res. Bd. Res. Rep. no. 6-B, 9–16 (1949).

Flow through a short tube is analyzed by dividing it into three segments: (1) the entrance segment in which the losses occur due to separation, (2) an intermediate segment in which the boundary layer grows from zero thickness until it fills the pipe, and (3) the final segment in which the velocity profile is fully established.

The effective pipe-friction factor for such a tube was calculated by using an inlet-loss coefficient for the first segment, by making boundary-layer calculations in the second segment, and from standard-friction factor tables for the third segment. The effective friction factors calculated by this method were checked with experimental data of flow through concrete and corrugated iron culverts obtained at the University of Iowa about 20 years ago. The calculations check very satisfactorily with experimental results. The effective friction factors are given for various lengths of pipe in tabular and curve form that are convenient for practical calculations. The author points out that the formulas proposed by the Iowa investigation are not correct dimensionally and cannot express the flow rates through conduits of varying lengths. By analyzing the Iowa tests, the author also derives a velocity-distribution law for corrugated iron pipes.

Vito A. Vanoni, USA

282. Curtis L. Larson, Experiments on flow through inlet gratings for street gutters, Highw. Res. Bd. Res. Rep. no. 6-B, 17–26 (1948).

Full-scale tests were conducted in the St. Anthony Falls Hydraulic Laboratory, University of Minnesota, on four types of inlet gratings for street gutters. Each inlet was tested over a range of flow velocities. The gutter had a constant cross slope of 20.6 to 1 in all tests. The tests show that the discharge capacity of gratings can be increased by allowing a small amount of flow, say from 0.1 to 0.2 cu ft per sec to be discharged over the grating and into the gutter downstream. Considerable increase in discharge capacity was effected for gratings with bars normal to the flow direction by rounding off the bars downstream from the leading edge. Tests of the relative effectiveness of the gratings in passing debris were also made in the laboratory. Sample calculations are given for the design of a typical gutter inlet, and a comple description of the behavior of the gratings is given.

Vito A. Vanori, USA

283. A. Craya, A theoretical study of the flow of superimposed liquid strata of different densities (in French), Houille blanche 4, 44–55 (Jan.-Feb. 1949).

The author considers two superimposed liquids of different densities, $\rho + \Delta\rho$ below and ρ above, flowing through a tank which is closed at one end by a vertical wall with a long, narrow slit at a height d above the interface; the liquid flows at the rate of q per unit length of slit. Using the Bernoulli and static equilibrium equations, he shows that the velocity of the upper liquid is the same as in a flow with a free surface, but with the gravity g reduced to $g\Delta\rho/\rho$ (new Froude number!). Then, neglecting viscosity and surface-tension effects, he applies dimensional analysis

and determines, in particular, a relation for the limiting distance h of the interface from the slit, at which no heavy liquid flows through the slit:

$$g\Delta\rho h^3 = \alpha\rho q^2.$$

After giving some interesting rough approximations to the constant α, the author finds by Richardson's method and conformal mapping that $\alpha = 0.438$, which fully agrees with experiment (see following review). The shape of the interface is obtained with the help of Legendre integrals. The author discusses the limitations of the results and deduces certain conditions under which the size of the slit may be neglected. The very interesting paper closes with an extension of the problem to three-dimensional flow and an estimate of viscosity effects.

Aladar Hollander, USA

284. Paul Gariel, An experimental study of the flow of superimposed liquid strata of different densities (in French), Houille blanche 4, 56–64 (Jan.-Feb. 1949).

To verify A. Craya's theoretical study (see preceding review) the author first made some equally resourceful experiments with two liquids of different salinities. The two liquids, the lighter above the heavier, flowed through a tank with glass side walls ending in a vertical wall with a slit a few mm high extending across the full width of 0.3 m. By coloring one of the liquids, he could see and photograph the interface and found it to be of the expected shape and quite stable. Its limiting height was very definite and confirmed the theoretical relation of the parameters, and the value of the constant, through a rather wide range of the individual variables. Similarly good agreement was found with the general equations of three-dimensional flow through a circular orifice.

Secondly, to get closer to the case of the flow of ocean water and verify Claude's expectation that he will get very cold water from a great depth without mixing with water at higher levels and temperatures, experiments were made with water of salinity continuously increasing with depth. These showed that, except for minor secondary currents, the main flow is from layers with definite boundaries at the general level of the orifice. While the theoretical study of the flow showed the expected dependence on two parameters, a study of the thickness of the flowing layer at a distance from the orifice and the flow rate for a given gradient of density, would require a third parameter (viscosity function) which is left for further study. Aladar Hollander, USA

285. Léopold Escande, A method for calculation of water-hammer effects (in French), C. R. Acad. Sci. Paris 228, no. 8, 653–656 (Feb. 1949).

Damaging water-hammer effects may occur in the line between a main gate valve, located upstream, and the distributor, located downstream close to the turbine. These effects take place if the upstream valve is closed before the downstream distributor valve. The interval of time between the two closing is most important. Whether the line is equipped with an air-inlet suction valve or a float valve, a graphical method of analysis is proposed and explained for various closure time intervals.

An additional article is to appear, describing the application of such a graphical method to an actual problem.

J. H. Mercier, USA

286. André Nizery, Note on bed-load movement (in French), Houille blanche no. spec. A, 365–372 (1949).

This is a review of papers on bed-load movement presented at the second congress of the International Association for Hydraulic Structures Research by Meyer-Peter and Muller, Tison, and Mavis and Laushey. The author summarizes the last results of the important research work carried out by Meyer-Peter and Muller (Zurich) and presents the evolution of their ideas. He also shows how the last formula of the Swiss authors can be interpreted when introducing the traction force of Du Boys. This last formula, when duly transformed in terms of the traction force, is comparable to the U.S. Waterways (Vicksburg) formula.

The second part of the paper summarizes some recent research work (Tison, Mavis and Laushey) on the limiting velocity producing no bed-load movement. Charles Jaeger, England

287. E. H. Buller and B. H. Ford, Hydraulics and the guided missile, CADO tech. Data Dig. 14, 17–23 (Apr. 1949).

The author presents a semipopularized picture of the role of hydraulics in guided missiles and pilotless aircraft. Hydraulic problems appear in the design of high-pressure and high-capacity fuel pumps, and high-performance control systems. Some difficult aspects of these problems are discussed in detail: design of proportional control; analysis and synthesis of computing circuits; the effect of time lag in the servo system; and design difficulties connected with small size equipment.

Andrew Vázsonyi, USA

288. M. Bachet, Graphical method for flood warnings (in French), Houille blanche no. spec. B, 720–727 (1948).

The study of the flood wave in a river, based on the continuity equation, shows that the displacement of the wave top consists of a propagation and an attenuation; both may be defined formally in a rigorous manner. For warning purposes the calculation should however be as fast as possible, and it is therefore necessary to introduce some simplifications. The author indicates a simple graphical method. Duilio Citrini, Italy

Incompressible Flow: Laminar; Viscous

(See also Revs. 281, 282, 283, 284, 337, 341, 352, 360, 363, 386)

289. L. G. Loytsiansky, Reaction of the boundary layer on the distribution of pressure over the surface of a body in a flow of real fluid, Hdqtrs. Air Mat. Comm. Dayton Transl. no. F-TS-1210-IA (GDAM A9-T-28), 21 pp. (Jan. 1949): see Rev. **482,** Mar. 1948.

290. George F. Hausmann, The theoretical induced deflection angle in cascades having wall boundary layers, J. aero. Sci. 15, 686–690 (Nov. 1948).

The author determines an approximate correction of deflection-angle data for cascade tunnels by considering the induced velocities of the modified vortex system associated with two-dimensional cascades having wall boundary layers. After replacing the cascade vanes by lifting lines, and assuming suitably curved end walls to simplify analysis, two correction factors are derived. One of these accounts for the induced deflection due to the trailing-vortex system in the wall boundary layer, the other for the decrease in total circulation of the bound-vortex system. The analysis is limited to small values of boundary-layer thickness. A turbulent seventh-root boundary-layer velocity distribution is assumed. Changes of the boundary layer in shape and thickness are neglected. Henry Görtler, Germany

291. G. Kreisel, Surface waves, Quart. appl. Math. 7, 21–44 (Apr. 1949).

This paper deals with surface waves in a two-dimensional medium of finite depth, obstacles being present either at the bottom or at the surface which are otherwise parallel. The usual

assumptions are made leading to Laplace's equation for the velocity potential and to a mixed boundary condition at the free surface; harmonic oscillations only are considered. The domain of the fluid is first transformed into an infinite strip, and the transformed problem is reduced to an integral equation of the second kind with infinite range. It is shown that the latter can be solved by iteration. The theory is applied in the case of a vertical barrier of arbitrary height in the bottom of the medium (canal), and to the case of a flat plate lying in the surface. Various analytical expressions, exact and approximate, are derived for the reflection coefficients, and numerical results are given for the second case. Some formulas relating to reflection at a low reef of finite width are also quoted without a detailed analysis. The paper is written with a view to applications to the design of breakwaters and to the oscillation of ships in waves. A high standard of mathematical rigor is maintained throughout.

[Compare the work by A. Weinstein, C. R. Acad. Sci. Paris 184 (1927); J. J. Stoker, Quart. appl. Math. 5 (1947); F. Ursell, Proc. Cambridge Phil. Soc. 43 (1947)]. A. Robinson, England

292. Hans Fromm, Laminar flow of Newtonian and Maxwellian fluids (in German), Z. angew. Math. Mech. 28, 43–54 (Feb. 1948).

The author considers flow of fluids satisfying the Maxwell type of stress-strain relations; i.e., the rate of deformation is expressible as a linear combination of the stress and the rate of change of stress (following the motion of an element of fluid). In particular, the stationary flows between two parallel plates and through a circular tube are considered. Two possible solutions are obtained for the distribution of velocity and pressure, one "stable" and the other "unstable." The stable solution is similar to the solution for the Newtonian fluid and approaches it in the limit of vanishing elasticity. The unstable solution has an infinite velocity along the axis of symmetry. Pressure measurements and the relation of the present theory to structural viscosity of colloidal solutions are discussed. C. C. Lin, USA

293. J. Kampé de Fériet, On the decrease of the kinetic energy of a viscous incompressible fluid occupying a finite volume bounded by fixed solid walls (in French), Ann. Soc. sci. Brux. Ser. I, 63, 36–45 (Mar. 3, 1949).

Let E be the kinetic energy of a viscous incompressible fluid enclosed by fixed walls. In the case of a plane domain M. Leray [J. Math. pures appl. 13, 331–418 (1934)] showed that E decreased with time at least as fast as an exponential function,

$$E \leqslant E_0 \exp(-\alpha t), \tag{1}$$

where α is a positive constant. In a preceding paper [C. R. Acad. Sc. Paris 223, 1096–1098 (1946)] the present author obtained the same result in a different way and with less restricting conditions, and gave an explicit expression for the constant α. The aim of the present paper is to establish the inequality (1) for a spatial domain of boundary S.

From the equations of motion one obtains an expression for dE/dt which consists of an integral extended over D; the integrand contains the first derivatives of the velocity components (see Lamb, *Hydrodynamics*, 6th ed., p. 580). The author uses a somewhat different form of this integral and a certain inequality, and finds that

$$E(t) \leqslant E(0) \exp(-6\pi^2 \nu \tau / L^2),$$

where $3L^{-2} = L_1^{-2} + L_2^{-2} + L_3^{-2}$ (L_1, L_2, L_3 are the sides of the minimum parallelepiped which contains $D + S$). A particular consequence of this inequality is that $E(t)$ decreases to zero when $t \to \infty$. Ratip Berker, Turkey

294. D. E. Dolidze, A nonlinear boundary problem of the unstable motion of a viscous incompressible fluid, Prikl. Mat. Mekh. 12, 165–180 (Mar.-Apr. 1948).

The author gives an existence and uniqueness theorem for incompressible, unsteady fluid flow satisfying the nonlinearized Navier equations. The velocity is given at all times inside of and on a sufficiently regular surface. Auxiliary partial differential equations are introduced, and their solutions permit the author to reduce the problem to twelve nonlinear integral equations, solvable by successive approximations. A local solution results. The author does not mention the global results of Leray [Acta Math. 63, 193–248 (1934)] or the work of Oseen.

A. W. Wundheiler, USA

295. D. Meksyn, Note on stability of laminar viscous flow between parallel planes, Proc. Roy. Soc. London Ser. A, 195, 174–179 (Dec. 1948).

The author shows that the transformation of the slowly varying integrals, obtained from his treatment of the equation of hydrodynamic stability, can be identified with those given earlier by Heisenberg and Tollmein, although the results appear in different forms. [Reviewer's remark: For the existence proof of solutions having the usually considered asymptotic representations, see W. Wasow, Ann. of Math. 49, no. 2, 852–871 (1948)].

C. C. Lin, USA

296. P. P. Kufarev, A solution of the boundary problem for an oil well in a circle (in Russian), Doklady Akad. Nauk SSSR 60, 1333–1334 (1948).

The author determines the exact flow of oil in a unit circle in the presence of a single eccentric well inside the circle. The solution is explicit in terms of a rational complex function. Ed.

297. D. T. MacRoberts, Effects of transient condition in gas reservoirs, J. petrol. Tech. 1, 36–38 (Feb. 1949).

The disturbance propagated into a gas reservoir due to a sudden change in pressure at a well is studied under the assumption that the pressure distribution set up is logarithmic. The time for the disturbance to reach distant points is calculated and found to be a function of the physical properties of the sand and geometry of the reservoir, as well as of the pressures in the system.

It is pointed out that the propagation velocity of the disturbance is practically finite, and therefore that practical transient times are much longer than commonly supposed. As a consequence, care must be exercised in the interpretation of experimental data taken on reservoirs under transient conditions.

W. G. Cornell, USA

298. V. M. Falkner and Sir Charles Darwin, The design of minimum drag tip fins, and **The design of minimum drag inboard fins,** Rep. Memo. aero. Res. Counc. Lond. nos. 2279 and 2280, 15 pp. each (1945, issued in 1949).

Starting with the premise (based on the method of Trefftz) induced drag is given by spanwise constant downwash on the wing and zero sidewash on the fin, the circulation distribution in the vortex sheets shed by the wing and fin is computed by use of conformal transformations. Configurations considered include a semispan of a wing with unsymmetric upper and lower tip fins, with the special cases of symmetric upper and lower tip fins and upper and lower tips fins alone; and a semispan of a wing with symmetric upper and lower inboard fins at an arbitrary spanwise location, with the special cases of upper and lower inboard fins alone.

The circulation distribution along wing and fins for minimum

induced drag is calculated for the various configurations, using a range of geometries. The procedure for the design of untwisted wings is discussed and examples are given. The effect of retracting a single inboard fin is discussed. W. G. Cornell, USA

299. **J. O. Hinze, Forced deformations of viscous liquid globules,** Appl. sci. Res. Sec. A., 1, no. 4, 263–272 (1949).

The author studies the deformations of liquid globules having a translatory motion relative to air, as may occur, for instance, during distintegration processes of high-speed liquid jets and for falling drops, the effect of viscosity being taken in consideration. The problem is restricted to slight deviations from the spherical shape and therefore the equations of the motion of a viscous incompressible fluid are simplified by omitting nonlinear terms. The boundary conditions at the surface of the globule, consistent with its translatory motion relative to air, consist of zero tangential stress and a prescribed external pressure distribution of rotational symmetry. The motion of the liquid is supposed to have the same rotational symmetry. Two solutions are obtained for the cases of slight and large viscosities. For the other cases no solutions are obtained; it is then necessary to solve a complicated auxiliary equation. In the case of slight viscosity the damping coefficient found by the author differs from that calculated by Lamb who uses a different method (*Hydrodynamics*, 6th ed., art. 355, New York, 1945), and reasons are given for which Lamb's result cannot be correct. Ratip Berker, Turkey

300. **J. O. Hinze, Critical speeds and sizes of liquid globules,** Appl. sci. Res. Sec. A, 1, no. 4, 273–288 (1949).

Splitting up of a liquid globule that has a translatory motion relative to air depends upon the interplay between the air resistance and the surface tension, and therefore upon Weber's number: the drop splits up if this number exceeds a critical value, which for falling drops is different from that for drops suddenly exposed to a flow of constant speed. The paper indicates a relation between the two cases, based on the consideration of the critical value of the deviation of the drop from spherical shape at the point of stagnation. The theory is based upon formulas, derived by the author in the preceding paper, for the slight deformation of a liquid globule, caused by normal forces on its surface (neglecting the effect of tangential stresses on deformation), and has therefore only a formal meaning; approximate expressions for the deviation as a function of Weber's number are derived for both situations and for very slight (i.e., the usual) or very large effect of the viscosity of the liquid. Recent experiences of Merrington and Richardson on falling drops have furnished a critical Weber's number of about 10; the corresponding value for the case of sudden exposure to an uniform flow should be about 6.
Duilio Citrini, Italy

301. **T. Bryn, Speed of rise of air bubbles in liquids,** David Taylor Model Basin Transl. no. 132, 7 pp. (June 1949).

Technique and results of an experimental investigation of the speed of rise of air bubbles in liquids are presented. Classification of bubbles as "large" or "small" was made on the basis of their behavior, the large bubbles becoming mushroom-shaped and the small bubbles remaining spherical. It was found possible to develop an empirical formula for the velocity of large bubbles dependent only upon the volume of the bubble and upon Laplace's constant for the liquid, independent of viscosity. Contrary to expectations, the resistance of small bubbles in certain liquids was considerably less than that of solid spheres, a phenomenon which may be explained by the assumption of partial and continuous replacement of the boundary layer while the bubble is rising.
John E. Goldberg, USA

Compressible Flow, Gas Dynamics

(See also Revs. 330, 335, 336, 365)

302. **K. P. Stanyukovich, Self-similar gas flows in a gravitational field** (in Russian), Doklady Akad. Nauk SSSR 64, 467–470 (June 1, 1949).

[See Rev. 309, this issue.] A certain type of self-similitude $[ut/r = f(rt^{-\alpha}), (p/\rho)(t^2/r^2) = \varphi(rt^{-\alpha}), \rho t^2 = \psi(rt^{-\alpha})]$ imposed on the unsteady motion of a gas with complete spherical symmetry under the influence of its own internal gravitational field. Several families of solutions are investigated, some of which may have important astrophysical applications. In the stationary case an equation of the Emden type is obtained, which has, in addition to the known solution for $C_P/C_V = k = 2$, a solution expressed as a quadrature for $k = 6/5$. Wallace D. Hayes, USA

303. **Hans Richter, The stability of a compression shock in a concave corner** (in German with Russian summary), Z. angew. Math. Mech. 28, 341–345 (1948).

The shock attached to the point of a wedge symmetrically placed in a supersonic stream has two mathematically possible configurations: the "strong" solution of high intensity, and the "weak" solution of lesser intensity. In practice only the weak solution is observed, and numerous explanations have been advanced to account for this preference. The author dismisses these and advances an argument to show that no grounds for distinction exist in the realm of stationary flows; that, in short, the question is one of stability under disturbance.

The author explains the stability of the weak solution and the instability of the strong one on the basis of the shock-polar diagram. His argument is roughly as follows: For the weak solution, an increase in shock strength entails a sharper turning of the streamlines through the shock, for the strong shock a less sharp deflection. When the attached shock is weak, a sudden increase of pressure behind it tends to separate the streamlines from the wall, an expansion wave is set up, and the pressure drops. A sudden drop in pressure, on the other hand, drives the streamlines into the wall, which tends to restore the pressure. For the strong configuration the reverse is true: pressure increases tend to build up until the shock detaches itself from the nose, pressure decreases continue until the stable weak configuration is attained. If for some reason only the strong stationary solution exists, this line of reasoning suggests a continuing oscillation of the shock between two nonstationary extremes, one attached, the other detached. If a narrow wedge is followed by a rectangle much broader than the base of the wedge this situation exists, and the resulting high frequency oscillations have been observed by Oswatitsch.
Courtesy of Mathematical Reviews D. P. Ling, USA

304. **Franz Wecken, Ramification of shock waves at reflection** (in German with Russian summary), Z. angew. Math. Mech. 28, 338–341 (1948).

When a shock wave falls obliquely on a wall, the primary and reflected waves may form a V with vertex on the wall and moving along it, or a Y configuration may result, abruptly or gradually, with the center of the Y moving forward and outward. The former case is always observed when the angle α between primary shock and wall is small, the latter (called Mach reflection, as opposed to "regular" reflection) when α is large. Cranz and Schardin suggested that regular or Mach reflection would take place according as the stream deflection determined by shock strength and incidence angle α was less than or greater than the maximum allowed by the shock polar diagram. On the other

hand, Schultz-Grunow advanced the criterion that the reflection would remain regular so long and only so long as the shock propagation velocity determined by the pressures in front of the primary and behind the reflected wave remained less than the velocity of the vertex along the wall. The author resolves this ambiguity by showing that the Cranz-Schardin criterion is correct for weak shocks, the Schultz-Grunow criterion for strong shocks. In the latter case the transition is continuous, while in the former it is abrupt.

Courtesy of Mathematical Reviews D. P. Ling, USA

305. V. G. Levich, The motion of gas bubbles at large Reynolds numbers (in Russian), Zh. exp. teor. fiz. 19, 18–24 (1949).

The resistance experienced by a very small gas bubble rising from buoyancy in a viscous liquid is known to be given quite accurately by Stokes's law for the resistance of a solid sphere in slow motion, the surface tension apparently causing the bubble to act like a solid sphere. The author wishes to work out an approximate theory for the case of large bubbles in a viscous fluid. For this purpose he develops a boundary-layer theory for the free surface as a perturbation of the corresponding inviscid fluid motion. In order to find the drag of a large bubble, he takes it spherical in form and computes the rate of dissipation of energy, assuming the solution for an inviscid fluid to hold in the viscous case. He obtains double Stokes's formula [this fact is also mentioned by Bateman on p. 157 of his article in the Report of the Committee on Hydrodynamics, National Research Council, Washington, D. C., 1932]. It is stated that preliminary experiments tend to confirm this value for the drag. Although the author discusses the limitations of this formula, it seems to the reviewer that they are much more serious than he supposes. In particular, the assumption of a spherical bubble seems far from the facts for bubble radii at which surface tension can be neglected. The alteration of the bubble shape caused by the combination of the hydrostatic pressure and the dynamic pressure is so extensive that any increase in drag could easily be attributed to increased form drag. There also appear to be some inadequacies in the development of the free surface boundary-layer theory.

J. V. Wehausen, USA

306. Max A. Heaslet, Harvard Lomax, and Arthur L. Jones, Volterra's solution of the wave equation as applied to three-dimensional supersonic airfoil problems, Nat. adv. Comm. Aero. Rep. no. 889, 22 pp. (1947).

See Rev. 499 (Mar. 1948).

307. W. Quade, On the theory of plane continuous gas waves of finite amplitude (in German), Z. angew. Math. Mech. **25/27**, 215–232 (Nov.-Dec. 1947).

The displacement of the gas particles is introduced as the dependent variable, and shown to satisfy the Monge-Ampère differential equation. The solution of the latter can be given in closed form for the isothermal and adiabatic cases. The well-known recursion edges appear in the integral surface of the displacement. Arthur Kantrowitz, USA

308. Charles Roumieu, Transient regimes in two-dimensional supersonic aerodynamics (in French), C. R. Acad. Sci. Paris 228, no. 1, 56–57 (Jan. 1949).

After writing down the well-known equations of supersonic unsteady flow, the author simply gives the formal expressions of the lift and pitching-moment coefficients in the case of a rigid airfoil, without writing them in full. Carlo Ferrari, Italy

309. K. P. Stanyukovich, Automodel plane and axisymmetric steady gas flows (in Russian), Doklady Akad. Nauk SSSR 64, 29–32 and 179–181 (Jan. 1949).

[Cf. Revs. 1, **1384**; 3, **111**.] A steady compressible flow is termed "automodel" if the velocity, density and pressure are functions of a single variable which is itself a function of the original independent variables. In the cases investigated the restriction to plane or axisymmetric flow is made at the outset. No a priori specification is made regarding irrotationality or isentropy.

The Prandtl-Meyer flow and the conical flow of Buseman [Z. ang. Math. Mech. 9 (1929)] are obtained as special cases. Some more general solutions are obtained in the form of unsolved differential equations. Finally, it is noted that for the most general case of steady spatial flows it is impossible to find automodel solutions which depend on only one parameter.

In the second paper, the author extends his automodel concept to unsteady plane and axisymmetric flows of a compressible fluid. As in the steady case, solutions of the assumed forms are obtained in the form of differential equations. As special cases the irrotational planar flows with point symmetry discussed by L. Sedov [title source, 47, no. 2 (1945)] and by the author [title source, 48, no. 5 (1945)] are found. Wallace D. Hayes, USA

310. Max A. Heaslet and Harvard Lomax, The application of Green's theorem to the solution of boundary-value problems in linearized supersonic wing theory, Nat. adv. Comm. Aero. tech. Note no. 1767, 41 pp. (Apr. 1949).

A summary is first given of the basic linearized equations of potential steady compressible three-dimensional flow and of potential unsteady two-dimensional flow. General solutions of these equations, satisfying general sets of boundary conditions, are then given by means of Green's equivalent layer of sources, sinks and doublets. Use is made here of Hadamard's technique of the "finite part" of an improper integral. For the study of unsteady flow, the analogy is used that the equation for unsteady two-dimensional flow can, by a simple change of variables, be reduced to exactly the same form as the three-dimensional equation for steady flow with a free-stream Mach number of $\sqrt{2}$.

Detailed applications of the general solutions are made to four cases. (1) The load distribution $\Delta p/q$ over a triangular yawed flat plate at an angle of attack is derived in terms of leading edge angle and sideslip angle. (2) The downwash is calculated behind an unyawed triangular flat plate at an angle of attack, and a plot is shown of downwash variation along the centerline of a 45-deg swept wing. (3) Loads and lift coefficient as functions of $t'(=a_0t/l)$ are derived for a wing which has been flying at supersonic speed and experiences an abrupt angle-of-attack change without pitching. Comparison is made with theoretical results obtained for growth of lift-curve slope with time in the case of subsonic flight. (4) Load distribution is derived over a sweptback lifting plate with $M_0 = 1$, in which case the basic differential equation reduces to the Laplace equation for two dimensions.

Figures illustrating numerical results are given throughout. The paper is clearly written and includes a very adequate list of references. M. Morduchow, USA

311. M. A. Biot, Transonic drag of an accelerated body, Quart. appl. Math. 7, 101–105 (Apr. 1949).

In the linearized theory the steady-state drag of a body becomes infinite at the speed of sound. This is not true, however, in unsteady motion. The author investigates this phenomenon by considering the uniformly accelerated motion of a two-dimensional wedge along its axis of symmetry.

The wedge is represented by a distribution of variable sources

along this axis. When during its accelerated motion the nose of the wedge reaches a certain point of the axis, a source must originate at that point, producing a sudden normal velocity which afterwards increases linearly with time. From this condition the source distribution and hence the pressure distribution are determined. When the wedge reaches the speed of sound the drag remains finite but tends to infinity as the acceleration decreases.

The author points out that by superposition of positive and negative wedges the method solves the two-dimensional linearized drag problem not only for a symmetric body of arbitrary shape with constant acceleration but may easily be generalized to cover the case of arbitrary motion. Heinz Parkus, Austria

312. **J. W. Craggs, The application of the hodograph method to problems of subsonic compressible flow in two dimensions,** Rep. Memo. aero. Res. Counc. Lond. no. 2273, 16 pp. (1946, publ. 1949).

A thorough discussion is provided of the linearization methods of two-dimensional, irrotational, compressible flows by transformations to the hodograph plane. The difficulties associated with the assignment of boundary conditions and the usual simplifications introduced to obtain solutions are discussed. The present paper, which is to be followed by a second investigation involving a more general discussion, uses the Kármán-Tsien approach in which the adiabatic equation of state is replaced by a linear function ($\gamma = -1$). Flows around closed cylinders are then discussed, both with and without circulation. Difficulties encountered in closing the body profile when circulation is present are overcome by an unpublished method ascribed to K. von Jaeckel. As an illustrative example, the flow for a symmetric Joukowsky airfoil is calculated at angles of incidence equal to zero and 2°52′ at a free-stream Mach number of 0.6. By these methods it is impossible to keep the airfoil shape fixed for the two angles of attack, but the difference is small.

Max A. Heaslet, USA

313. **B. Göthert and K. H. Kawalki, The compressible flow past various plane profiles near sonic velocity,** Nat. adv. Comm. Aero. tech. Memo. no. 1203, 51 pp. (Mar. 1949) (transl. from ZWB, UM no. 1471, 1945).

Calculations and results are presented for various subsonic and transonic two-dimensional flows using the "single-source method." In this method the compressible flow field is considered as though it were an incompressible field with a suitable source distribution which takes care of the change in the continuity condition due to compressibility. The source distribution is approximated by a number of suitably placed point sources.

The results include cases with locally supersonic regions. In each case, as the free-stream Mach number is increased, a point is reached beyond which the calculations of the method do not converge in the required manner. This is interpreted as indicating the point beyond which shockless symmetric flow may not exist.

Wallace D. Hayes, USA

314. **G. Drougge, The flow around conical tips in the upper transonic range,** Flygtekn. Forsoksanst. Medd. no. 25, 24 pp. (1948).

The author investigates theoretically and experimentally the pressure distributions over two cones with semiangles of 41.5 and 45 deg. The experiments were made at Mach numbers 1.40, 1.60, 1.83 and 2.08. The computations were carried out at Mach numbers 1.80 and 2.15 for the larger cone. The Mach numbers for which the shock detaches are 2.10 and 2.38, respectively, for the two cones. The method of computation follows a procedure given by Maccoll. One starts with a guess for the location of the shock and the sonic line and then finds the subsonic field in the vicinity of the cone tip by means of the relaxation method. If the result does not satisfy the shock conditions and the conditions at the sonic line, the shape of the shock and of the sonic line is changed. The agreement between theory and experiment is good.

The reviewer does not see how the fact that the shoulder of the body is a corner, enters the computation. Judging from the results, one obtains the impression that this condition, which is certainly of physical importance, has been overlooked.

Gottfried Guderley, USA

315. **Antonio Ferri, Application of the method of characteristics to supersonic rotational flow,** Nat. adv. Comm. Aero. Rep. no. 841, 12 pp. (1946, issued in 1949).

A procedure for applying the method of characteristics to the calculation of two-dimensional and of axisymmetric supersonic flows with vorticity is discussed. It is then used to compute the external flow and the supersonic portions of the internal flow (including the shock shape) about an open-nosed body of revolution, and the result of several examples are given. The case of a supersonic inlet with a central body is also considered.

J. S. Isenberg, USA

316. **H. F. Ludloff and F. Reiche, Linearized treatment of supersonic flow through ducts,** J. aero. Sci. 16, 5–21 (Jan. 1948).

The authors have obtained some special solutions to the linearized supersonic flow equations for bodies of revolution with particular attention to the interior region. Thus examples of the flow in ducts are obtained. Although more general results may be obtained by superposition, the method does not lead directly to a solution of the general problem.

One general result appears from the investigation: A compression wave of discontinuous type initiated at the tip or wall of the duct becomes a strong compression followed by an expansion (of logarithmic type) after the wave passed the axis onto the aft branch of the characteristic cone. Wallace D. Hayes, USA

317. **J. Lacaine, Short diffusers for wind tunnel** (in French), Rech. aéro. Paris no. 7, 35–40 (Jan.-Feb. 1949).

A series of slotted diffusers composed of frusta of 7 and 30-deg cones were tested in a 12-in. open circuit tunnel of the Eiffel type at about 600 fps The tunnel coefficient was determined for a dozen combinations differing in length and in the number and arrangement of slots. Also, the static pressure distribution in the test section was measured. J. S. Isenberg, USA

318. **T. Y. Thomas, The consistency relations for shock waves,** J. Math. Phys. 28, 62–90 (Apr. 1949).

This is a continuation of two previous papers by the author [same source 26, 62–68 (1947); 27, 279–297 (1949); Revs. 1, 147 (Jan. 1948); 2, 1294 (Oct. 1949)]. The situation considered is that of a pointed symmetric obstacle in a uniform supersonic stream with an attached curved shock. The oncoming flow has Mach number μ and makes an angle $\alpha(P)$ with the shock direction at the point P of the shock. Let s be arc length along the shock line, η arc length along any streamline behind the shock; k and K are the curvatures of the shock and of the streamline at the shock. The author then shows that under appropriate conditions of differentiability there exist "consistency relations" of the form

$$d^nK/d\eta^n = G_n(M, \alpha)d^nk/ds^n + H_n$$

for $n = 0, 1, 2, \ldots$, where H_n is a polynomial in k and its derivatives of order less than n. Here G_n and the coefficients of the

terms of H_n are rational functions of M^2 whose coefficients are functions of α. These relations are particularly interesting at the vertex V of the obstacle, where the left members are determined by the geometry of the obstacle. Then there will in general exist for a given M "singular" values of α(the initial shock direction) for which some $G_n(M, \alpha)$ vanishes, and for which the corresponding k_n is consequently indeterminate.

In the first of the two papers cited above, the first derivatives of the hydrodynamical variables were evaluated behind the shock, and from these an explicit expression for G_0 was derived (H_0 being zero). In the second, this function $G_0(M, \alpha)$ was tabulated and graphed against α for various values of M. In the present paper, the second derivatives are obtained, and the function $G_n(M, \alpha)$ explicitly presented. Tables and graphs are also given, exhibiting the singular values of α (two in number) for each M considered.

Courtesy of Mathematical Reviews D. P. Ling, USA

319. M. Schaefer, Connection between wall curvature and shock wave curvature in two-dimensional gas flow, Hdqtrs. Air Mat. Comm. Dayton Transl. no. F-TS-1202-1A, 22 pp. (1948).

This translation of a 1942 German paper is concerned with flow conditions which prevail immediately downstream of a curved shock wave attached to the nose of a pointed two-dimensional body in a uniform oncoming supersonic stream. In particular, a relationship is derived between the curvature of the body profile at the nose and the curvature of the shock wave at the same point. Expressions are also given for the local gradient of the velocity and pressure both along the shock wave and in the direction of the body surface. The derivation of the equations is carried out by means of two different methods which yield identical results. The first proceeds from the characteristic differential equations for purely supersonic flow. The second is based upon the general differential equations for compressible flow and is thus valid for conditions of subsonic as well as supersonic flow behind the shock wave. The final relationship between the two curvatures is shown to agree with the earlier result of Crocco [L'Aerotechnica 17, 519–536 (1937)]. Essentially similar results are to be found in the recent work of Thomas [J. Math. Phys. 26, 62–68 (1947)), Rev. **141** (Jan. 1948); Lin and Rubinov (J. Math. Phys. 27, 105–129 (1948)), Rev. **1391** (Sept. 1948); and Munk and Prim (J. Aero. Sci. 15, 691–695 (1948)), Rev. **1665** (Nov. 1948)].

Walter Vincenti, USA

320. Barrett S. Baldwin, Jr., Triangular wings cambered and twisted to support specified distributions of lift at supersonic speeds, Nat. adv. Comm. Aero. tech. Note no. 1816, 30 pp. (Feb. 1949).

The linearized conical flow theory of Busemann [same source, no. 1100 (1947)] has been applied to the calculation of camber and twist required for various lift distributions on a triangular (conical) wing with subsonic leading edges.

The function $f(\zeta)$ of the Busemann theory is taken in the form

$$f(\zeta) = iB\left[a_0 \log \frac{\zeta + m}{\eta - m} + {}_n\Sigma_1^{\infty}\, b_{2n-1}\,[\zeta/m - (\zeta^2/m^2 - 1)^{1/2}]^{2n-1}\right]$$

which by suitable choice of the b_k, a_0, and B can be made to represent arbitrary lift distributions. In particular the wing giving finite lift at a subsonic leading edge is found, as well as one which gives approximately constant lift across the span without having an indefinitely large local angle of attack at the center line.

William H. Pell, USA

321. Arthur L. Jones and Alberta Alksne, The yawing moment due to sideslip of triangular, trapezoidal, and related plan forms in supersonic flow, Nat. adv. Comm. Aero. tech. Note no. 1850, 65 pp. (Apr. 1949).

The yawing-moment coefficients (C_n) due to a finite sideslip angle (β) in supersonic flight are derived from linearized potential theory, including the effect of the theoretical leading-edge suction for wings having a subsonic leading edge. The theoretical computations are made for rectangular and trapezoidal wings of small and large aspect ratio, triangular wings with subsonic and supersonic leading edges, and sweptback arrow and delta wings. All plan forms are considered as flat plates with no dihedral. The results are based on the stability axis generally used in dynamic-stability computations.

It is realized that the full theoretical leading-edge suction force probably is not attained on the subsonic leading edge of a typical supersonic wing since it has either a sharp nose or one with very small nose radius. However the complete theoretical suction force is used because the actual force depends primarily on nonlinear viscosity effects which are unpredictable at present. If the leading-edge suction is entirely neglected then the yawing moment on a flat plate at small angles of attack is given by the rolling moment times the angle of attack ($C_{n_\beta} \approx -\alpha C_{l_\beta}$). This provides the lower limit to the probable value of the yawing moment due to sideslip. E. V. Laitone, USA

322. Kenneth Margolis, Effect of thickness on the lateral force and yawing moment of a sideslipping delta wing at supersonic speeds, Nat. adv. Comm. Aero. tech. Note no. 1798, 24 pp. (Jan. 1949).

A delta wing of rhombic cross section at zero lift is considered at small angles of yaw. The pressure distribution is calculated using the results of previous source-distribution methods, assuming that the Mach lines are still parallel to the unyawed lines. The essential revision in the unyawed pressure distribution then enters in the slightly changed source strengths. The possible error in this approximation is discussed.

The pressure distributions are integrated to obtain the side force and yawing-moment derivatives as a function of Mach number and aspect ratio. For wings of reasonable thickness these are found to be very small compared with effects that might be expected with a vertical tail surface. A. E. Puckett, USA

323. Bolesław Szczeniowski, Theoretical study of ram-jet propulsion, Canad. J. Res. Sec. A 26, 327–358 (Nov. 1948).

The author analyzes the ideal ramjet cycle with the aid of the entropy diagram, to obtain the relations between the energy released and the efficiency and thrust obtainable. Ideal gas flow is assumed, and the effects of heat and aerodynamic losses and shock waves are disregarded. General conclusions are drawn as to the optimum shape of the ideal engine from the standpoint of efficiency.

It is concluded that: (1) For highest efficiency, the air should initially be subjected to as high a dynamic compression as possible, after which a combustion chamber of constant cross section is used, and finally a nozzle. (2) If no dynamic compression by means of a diffuser is applied, it is impossible to obtain a positive thrust with a combustion chamber of constant cross section.

A. D. Kafadar, USA

324. Theodore von Kármán, Theoretical remarks on thrust augmentation, Reissner Anniv. Vol., 461–468, J. W. Edwards, Ann Arbor, 1949, $6.50.

The simplest case of thrust augmentation is considered; i.e., a jet of a given cross section injected into a cylindrical tube with an

entrance nozzle on the upstream side. Defining the thrust augmentation ratio as the ratio of the actual thrust to the product of the mass and velocity of the jet (instead of to its total impulse), the author shows that the augmentation ratio can be considerably larger if the pressure and velocity distributions at the mixing section are nonuniform than if uniform distributions prevail. The author believes that this may explain the high efficiency of certain augmentation devices in which the primary flow is introduced near the curved walls of the entrance nozzle.

A. D. Kafadar, USA

325. A. G. Gaydon and H. G. Wolfhard, The influence of diffusion on flame propagation, Proc. roy. Soc. Lond. Ser. A, 1044, 105–113 (Feb. 22, 1949).

This is another contribution to the attempts to understand the mechanism of flame propagation. The present paper further adds to the qualitative evidence that thermal effects alone are not sufficient to define the speed of a combustion flame in a combustible mixture.

After a review of the Damkohler thermal theory of flame speed, the authors present experimental results on the thickness of the luminous portion of the flame front, and relate it by very rough calculations to the time of passage of molecules through the reaction zone and the estimated corresponding diffusion distance for hydrogen atoms. This semiempirical relation between diffusion distance and reaction-zone thickness is used to deduce another form of flame-speed equation.

The equation obtained is better than that of earlier workers in that it predicts a flame speed independent of pressure. This, they report, is in good agreement with experiment for the hydrocarbons tested. Howard W. Emmons, USA

326. Bernard Wiener, Ambient pressure determination at high altitudes by use of free-molecule theory, Nat. adv. Comm. Aero. tech. Note no. 1821, 12 pp. (Mar. 1949).

Calculations of the pressure measured by an orifice according to free-molecule theory are presented and compared with similar results from continuum theory. Arthur Kantrowitz, USA

327. A. Fage and R. F. Sargent, Effect on aerofoil drag of boundary-layer suction behind a shock wave, Rep. Memo. aero. Res. Counc. Lond. no. 1913, 22 pp. (1943, publ. 1949).

This is a careful analysis of wartime two-dimensional tunnel experiments with rather limited equipment (2-in. airfoil chord; single fixed suction slot; inadequate optical and traverse information about the boundary layer). The reported effect of suction is similar to that for low speeds: a 40% to 50% reduction of drag-power which would be approximately compensated by power absorbed by the compressor. There was little effect on the "critical" Mach number. The flow appears to have been separated behind the shock wave in only one analyzed case so that the possibility of increased effectiveness of boundary-layer control when separation is present has not been truly tested. M. V. Morkovin, USA

328. W. F. Hilton and P. J. Wingham, Surface temperatures on an aerofoil at subsonic speeds, Rep. Memo. aero. Res. Counc. Lond. no. 2230, 13 pp. (1946, publ. 1949).

Measurements were made of the temperature distribution on the surface of an unheated airfoil in two-dimensional flow. The range of data covers Reynolds numbers 0.4 to 0.7 $\times$ 10^6, Mach numbers 0 to 0.75, and angle of attack 0 to 6 deg. The model had an aspect ratio 6 and low thermal conductivity. An observed inconsistency was found of about 10% in dimensionless temperature rise (ratio of rise of surface temperature above free-stream static temperature to rise of free-stream total temperature above free-stream static temperature). It is suggested that the inconsistency is due to the use of undried air and finite thermal conductivity of the model. However, comparison is made with theory [Squire, same source, no. 1984 (1942)]. At a Mach number deduced theoretically to be near critical, the surface aft of the shock wave was found to be abnormally warm. Further increase of Mach number showed a return to normal temperatures.

W. G. Cornell, USA

Turbulence, Boundary Layer, etc.

(See also Revs. 220, 327, 354, 365, 368, 393)

329. E. Abody, Friction in variable density fluid (in English), Publ. tech. Univ. Budapest (Müegyetemi Közl.) no. 3, 133–139 (1948).

Simple considerations of molecular motion are used to show that the viscous stress in a laminar motion in which the mass velocity μ and density ρ vary only in the y direction has the (approximate) form $\tau = \nu d(\rho u)/dy$ where ν is the kinematic viscosity. As an illustrative example of the use of this relation, the author considers the problem of two parallel streams, having the same velocity u_0 but different densities, which are separated along the negative x-axis and mix along the positive x-axis. The distributions of density and velocity components u and v across the mixing zone are found, using the assumption that ν has the same value in both streams. G. K. Batchelor, England

330. Coleman duP. Donaldson and Roger D. Sullivan, The effect of wall friction on the strength of shock waves in tubes and hydraulic jumps in channels, Nat. adv. Comm. Aero. tech. Note no. 1942, 36 pp. (Sept. 1949).

The attenuation of a shock wave due to the boundary layer in a tube of uniform circular cross section is found under certain assumptions. The authors assume that the thickness of the boundary layer at any point behind the shock depends only upon the time interval from the passage of the shock wave. The variation of the velocity in the stream direction is thus neglected. The shock wave starts a mass flow. The attenuation of the shock wave in a shock tube is obtained by setting this mass flow equal to that crossing the tube at the position of the diaphragm particles (which were initially at the starting position of the shock wave). Because of the familiar analogy, the theory is also developed for the attenuation of a hydraulic jump moving in a channel. Experimental results for both a shock tube and a hydraulic channel are presented. S. S. Shu, USA

331. Georg Hamel, A moderately turbulent motion (in German), Z. angew. Math. Mech. 28, 65–72 (1948).

The author considers a turbulent motion satisfying the following conditions: (1) it is a two-dimensional stationary motion of an incompressible viscous fluid in the region

$$-\infty < x < +\infty, \quad -h \leqq y \leqq h;$$

(2) the velocity components and the vorticity remain finite; (3) the x-component of velocity is nowhere zero. He then introduces a turbulent function which has the constant boundary value unity and is larger than unity elsewhere except when the flow is laminar. The turbulence function and the velocity distribution are related to each other. The relation between this work and (time-dependent) turbulent motion in the usual sense is not discussed.

C. C. Lin, USA

332. G. K. Batchelor, The role of big eddies in homogeneous turbulence, Proc. roy. Soc. Lond. Ser. A, 193, 513–532 (Feb. 1949).

The author investigates theoretically the role of fluctuations of small wave number (large eddies) in an infinite homogeneous turbulent field for an incompressible fluid. On the basis of Heisenberg's work [same source, 195, 402 (1948); Rev. 2, **774**] and conversations between Heisenberg, Townsend and the author, the following postulates, based on indirect experimental evidence, form the background of the present work: (1) Statistical equilibrium with time does not extend to the largest eddies; (2) these eddies contain only a small proportion of the total turbulent energy. Following these ideas it is postulated that a quasi-equilibrium exists among all except the large eddies. In order to investigate the role of the large eddies the author introduces a spectrum tensor, defined as the three-dimensional Fourier transform of the double velocity correlation tensor:

$$\Gamma_i^{\ k}(\mathbf{k}) = (2\pi)^{-3} \int\int\int R_i^{\ j}(\mathbf{r}) e^{\iota(\mathbf{k}\cdot\mathbf{r})} d\tau(\mathbf{r})$$

where $\mathbf{k}$ is the vector wave number,

$$\iota = (-1)^{1/2} R_i^{\ j}(\mathbf{r}) = \overline{u_i u_j}$$

denotes the correlation between the i component of the velocity at a point P and the j component at P', situated at a vectorial distance $\mathbf{r}$ from P, $d\tau(\mathbf{r})$ is a volume element at P'.

The role of small wave numbers is investigated by expanding $\Gamma_i^{\ k}(\mathbf{k})$ as a power series in the components of $\mathbf{k}$. Postulate (2) above is proved in that at small values of $\mathbf{k}$ the energy is proportional to $\mathbf{k}^4$. On the basis of the Navier-Stokes equations it is shown that the effect of pressure variations is to transfer energy between different velocity components with the same wave number, whereas the inertia effect is a flow of energy between different wave numbers for the same velocity component. By considering the series expansion for $\Gamma_i^{\ k}(\mathbf{k})$ it is shown that, for small wave number, all three processes: viscous decay, the tendency to isotropy due to the pressure term, and the transfer of energy between different wave numbers due to the inertia term, are negligible. The biggest eddies are therefore unchanged from their moment of creation; specifically any anisotropy which they possess at the beginning is permanent. Some measurements by Townsend in the final period of decay, in which the energy in the large eddies is an appreciable part of the total energy, indicate that the large eddies produced by a square-mesh grid are isotropic. The analysis is applied to a brief discussion of similarity parameters.

A. M. Kuethe, USA

333. A. M. Obukhov, The pulsation of pressure in a turbulent flow (in Russian), Doklady Akad. Nauk SSSR 66, 17–20 (1949).

In this paper the author presents a sufficiently general method for the determination of the structure function of the pressure field in an isotropic turbulent flow of incompressible fluid. Under the assumption of statistical homogeneity and isotropy of the field of velocities and the pressure field, and under the further assumption that the fourth moments of the derivatives of the field of velocities have approximately the same relation to the second moments as in normal distributions [see Millionshchikov, Bull. Acad. Sci. URSS Sér. Géograph. Géophys. 1941, 433–446], the equation

$$\Delta^2 \Pi(\rho) = -\sigma^2 (\partial^2 D^{\beta\delta}/\partial\xi^\alpha \partial\xi^\gamma)(\partial^2 D^{\alpha\gamma}/\partial\xi^\beta \partial\xi^\delta)$$

connecting the structure function of the pressure field

$$\Pi(\rho) = (p(M_2) - p(M_1))^2$$

and the structure function of the field of velocities

$$D^{\alpha\beta}(\xi) = [v^\alpha(M_2) - v^\alpha(M_1)(v^\beta(M_2) - v^\beta(M_1)]$$

is set up. The structure tensor for the locally isotropic field of velocities [see Kolmogoroff, C.R. Akad. Sci. URSS (N.S.) 30, 301–305 (1941)] has the form

$$D^{\alpha\beta}(\xi) = D_{nn}(\rho)\delta^{\alpha\beta} + \rho^{-2}(D_{nn} - D_{ll})\xi^\alpha \xi^\beta,$$

were D_{nn} and D_{ll} are the longitudinal and the transversal structure functions of the field of velocities, ρ is the distance between the points M_1 and M_2, and σ is the density of the medium. A particular case, $D_{ll} = b^2\rho^{2/3}$, $D_{nn} = {}^4/_3 b^2 \rho^{2/3}$ (b a constant) is considered. This leads to the relation $\Pi(\rho) = \sigma^2 D_{ll}^2(\rho)$ [see Kolmogoroff, loc. cit.; C.R. Acad. Sci. URSS (N.S.) 32, 16–18 (1941); Obukhov, C.R. Acad. Sci. URSS (N.S.) 32, 19–21 (1941); Bull. Acad. Sci. URSS Sér. Géograph. Géophys. 1941, 453–466]. The author points out that the pulsation of pressure in a turbulent flow apparently is connected with the problem of interaction between the field of sound and the turbulence; however, from the acoustical point of view of a turbulent flow, not only the spatial structure, but also the time spectrum of the pulsation of pressure may be of interest.

Courtesy of Mathematical Reviews E. Leimanis, Canada

334. Léon Agostini, On some properties of the total correlation function (in French), C. R. Acad. Sci. Paris 228, no. 10, 810–811 (Mar. 1949).

If $R_{ij}(\mathbf{r})$ is the correlation between the i and j components of turbulent velocity fluctuations at two points separated by the vector $\mathbf{r}$, the total correlation is defined as $R(\mathbf{r}) = {}_i\Sigma_3 R_{ii}(\mathbf{r})$. The author discusses a few of the formal properties of R and points out that in homogeneous turbulence the dynamical equation for R is of the heat-conduction type in three dimensions, the triple correlation term playing the part (in a loose sense) of a convection.

G. K. Batchelor, England

Aerodynamics of Flight; Wind Forces

(**See also Revs. 257, 287, 298, 306, 310, 320, 321, 322, 327, 353, 355, 377**)

335. Leonard M. Rose and John M. Altman, Low-speed experimental investigation of a thin, faired, double-wedge airfoil section with nose and trailing-edge flaps, Nat. adv. Comm. Aero. tech. Note no. 1934, 24 pp. (Aug. 1949).

The thin, sharp-edged airfoils used on supersonic aircraft generally develop low maximum-lift coefficients and are subject to large variations of profile drag with lift, thus directly affecting safety at the relatively low speeds necessary, for example, during take-off or landing. The present paper reports results of wind-tunnel tests to determine the effectiveness of nose and trailing-edge flaps toward improving section coefficients of one such airfoil. An increase of maximum section lift coefficient from 0.84 for the basic airfoil to 1.96 for the airfoil with both nose and trailing-edge flaps deflected is reported.

John E. Goldberg, USA

336. Charles W. Mathews and Jim Rogers Thompson, Comparative drag measurements at transonic speeds of rectangular and sweptback NACA 65-009 airfoils mounted on a freely falling body, Nat. adv. Comm. Aero. tech. Note no. 1969, 12 pp. (1949).

Results of experimental measurements of the drag of a rectangular wing and of an otherwise similar sweptback wing are presented and compared. Both airfoils were mounted on a freely falling body and data were obtained over a range of Mach numbers from 0.90 to 1.27. The sweptback plan form incorporated a sweep angle of 45 deg. The data indicate that the

drag of the sweptback wing was less than 30% of the drag of the rectangular wing at a Mach number of 1.00; this ratio rising to 40% at a Mach number of 1.20. Ed.

337. **Ira H. Abbott, Albert E. von Doenhoff, and Louis S. Stivers, Jr., Summary of airfoil data,** Nat. adv. Comm. Aero. Rep. no. 824, 264 (1945, issued in 1949).

This report presents data on the aerodynamic characteristics of NACA airfoils. The historical development of NACA airfoils is reviewed, and the general method used to derive the basic forms of certain series of these airfoils is given together with methods for obtaining approximate pressure distributions. The report includes an analysis of the lift, drag, pitching-moment and critical-speed characteristics of the airfoils and a discussion of the effects of surface irregularities. Data on high-lift devices are presented. Problems associated with lateral-control devices, leading-edge air intakes, and interference are briefly discussed. Application of the data to wing design and selection of root and tip sections are also briefly discussed. The methods used to obtain data in the Langley two-dimensional low-turbulence tunnels are presented in an appendix. Ed.

338. **Laurence K. Loftin, Jr. and Hamilton A. Smith, Aerodynamic characteristics of 15 NACA airfoil sections at seven Reynolds numbers,** Nat. adv. Comm. Aero. tech. Note no. 1945, 75 pp. (Oct. 1949).

Results are presented of investigations of the two-dimensional characteristics of 10 NACA 6-digit-series airfoils and 5 NACA 4- and 5-digit-series airfoils at Reynolds numbers from 9.0×10^6 to 0.7×10^6. Tests were made with smooth-surfaced airfoils and, except for pitching moment measurements, were repeated with standard roughness applied to the leading edge. Ed.

339. **Lawrence A. Clousing, William N. Turner and L. Steward Rolls, Measurements in flight of the pressure distribution on the right wing of a pursuit-type airplane at several values of Mach number,** Nat. adv. Comm. Aero. Rep. no. 859, 23 pp. (1946, issued in 1949).

Complete pressure distribution measurements were made on the right wing of a propeller-driven pursuit airplane. At five spanwise stations, upper and lower surface pressures were obtained to a Mach number of 0.8. In addition, internal wing pressures were determined. For this airplane, the proportion of the total lift carried by the wing varied considerably with Mach number, thus changing the wing-root bending independent of spanwise center-of-pressure shifts. No large outboard shifts of center of pressure were noted for this airplane in the Mach-number range tested, although the thickness ratio was less at the tip than at the root. However, the results obtained would be expected on the basis of critical Mach number of the tip and root sections (critical Mach number is defined as free-flight Mach number for local sonic velocity). At every section the Mach number for pitching-moment divergence exceeds the critical Mach number. The lift-curve slope increased with Mach number up to values of Mach number which were in some instances well above the critical. The wing internal pressures were small and negative.

It was noted that power had a significant effect on the measured pressures so that care has to be exercised in applying the results to jet-driven aircraft. Conrad A. Lau, USA

340. **James C. Sivells and Robert H. Neely, Method for calculating wing characteristics by lifting-line theory using nonlinear section lift data,** Nat. adv. Comm. Aero. Rep. no. 865, 19 pp. (1947, publ. 1949).

The purpose of this paper is to combine the contributions of Multhopp and several other writers, together with some original work, into a single complete method of calculating the lift distributions and force and moment characteristics of wings, using nonlinear section-lift data. Simplified computing forms are given for the calculation of symmetric lift distributions, and their use is illustrated by a detailed example. The adaptation of the method for use with linear section-lift data is also illustrated and comparison made with experimental results. Multhopp's systems of multipliers are employed to obtain the induced angle of attack directly from the spanwise lift distribution. Equations are developed for obtaining these multipliers for any even number of spanwise stations, and values are tabulated for 10 stations along the semispan for asymmetric, symmetric, and antisymmetric lift distributions. In order to minimize the computing time and to illustrate the procedures involved, simplified computing forms containing detailed examples are given for symmetric lift distributions. Similar forms for asymmetric and antisymmetric lift distributions, although not shown, can be readily constructed in the same manner as those given. The adaptation of the method for use with linear section-lift data is also illustrated. This adaptation has been found to require less computing time than most existing methods.

The wing characteristics calculated from general nonlinear section lift data have been found to agree much closer with experimental data in the region of maximum lift coefficient than those calculated on the assumption of linear section-lift curves. The calculations are subject to the limitations of lifting-line theory and should not be expected to give accurate results for wings of low aspect ratio and large amounts of sweep.

E. Arthur Bonney, USA

341. **Doris Cohen, A method for determining the camber and twist of a surface to support a given distribution of lift, with applications to the load over a sweptback wing,** Nat. adv. Comm. Aero. Rep. no. 826, 10 pp. (1945, issued in 1949).

A method is presented for calculating the camber and twist of an airfoil of arbitrary plan form, having an arbitrary spanwise and chordwise lift distribution. The lifting surface and its wake are replaced by equivalent horseshoe vortices lying in a plane.

The downwash at a point on the wing surface is calculated by application of the Biot-Savart relation for the summation of the contributions of all vortices. The ordinarily complicated expression is reduced to a single integral by a graphical procedure based upon a system of plane polar coordinates with origin at the point in question. Thus, having the equivalent vortex system, a single graphical integration yields the downwash at a chosen point. The function whose integral must be taken is represented as a curve, each point of which is determined by a single graphical integration.

Once the downwash has been calculated at a point, the slope of the wing surface is at once given as the ratio of the downwash to the free-stream velocity. Having enough points, the camber and twist of the wing are determined.

The method is applied to three wings. Three lift distributions were chosen, corresponding, according to two-dimensional theory, to: An elliptical wing of aspect ratio 6 and sweepback 30 deg, and two unswept, elliptical wings of aspect ratios 6 and 3. The more exact wing geometry is then found. It is concluded that two-dimensional theory is inadequate for sweptback wings and in the tip regions of unswept wings.

The inverse problem, to find the lift distribution on a chosen wing, is discussed. An iterative procedure is used to calculate the lift distribution on an elliptical wing with aspect ratio 6, sweepback 30 deg, and zero camber and twist.

W. G. Cornell, USA

342. Langley Research Staff, Summary of lateral-control research, compiled by Thomas A. Toll, Nat. adv. Comm. Aero. Rep. no. 868, 74 pp. (1947).

A summary is presented of available information on lateral control, covering mainly developments during the period from 1937 to 1946. Criteria used in lateral-control specifications, factors involved in obtaining satisfactory control, and methods used in making the experimental investigations are discussed. Much of the data on flap-type ailerons with various types of aerodynamic balance and on spoiler devices are presented in a form convenient for use in design. The effects of Mach number, boundary layer, and wing distortion are analyzed. Ed.

343. S. B. Gates and H. M. Lyon, A continuation of longitudinal stability and control analysis. Part I: General theory. Part II: Interpretation of flight tests, Rep. Memo. aero. Res. Counc. Lond. no. 2027, 34 pp. and no. 2028, 41 pp. (1944, publ. 1949).

The conventional theory of longitudinal stability and control is extended to include the partial differential coefficients $\partial C_L/\partial V$, $\partial C_d/\partial V$, $\partial C_m/\partial V$, and $\partial C_H/\partial V$ where V is the forward velocity. Such terms may arise because of: the distortion of the air flow at high speeds, the distortion of the structure under load, the effect of slipstreams or jets, or because of weights or springs in the elevator control system. Three cases are analyzed: (1) the static margin; (2) the maneuver point; and (3) the relation of (1) and (2) to the roots of the stability quartic. Both stick-fixed and stick-free conditions are considered. The assumptions underlying the theory of the maneuver point are explored, and the conditions under which it is valid are delineated. It is concluded that the stability and handling qualities depend mainly upon the static and maneuver margin, where the static margin does not, in general, coincide with the c.g. margin because of the inclusion of the partial differential coefficients above. It is shown that dangerous dynamic instability occurs for negative maneuver margins, but that only a slow divergence occurs for even large negative static margins as long as the maneuver margin is positive.

In part II the results of the general theory are applied to the analysis of flight-test results. The necessary measurements are described, a complete test program outlined and a method of analysis is given in some detail. Both static stability and maneuverability characteristics, stick-fixed and stick-free, are covered. In the absence of enough flight data for diagnosis of stability and control difficulties, a method for synthesizing distorted trim and stick-force-per-g curves is suggested. Examples of such syntheses are given for the following conditions: the tail plane becomes ineffective, the elevator becomes ineffective, the elevator tends to overbalance, the elevator becomes very heavy, the tab becomes ineffective, and the whole tail unit becomes ineffective. Curves are presented to illustrate the results, which are also summarized in convenient tabular form.

J. M. Wild, USA

344. H. B. Dickinson, Flight and tunnel test research on boundary-layer control, J. Aero. Sci. 16, 243–251 (Apr. 1949).

This report described an interesting series of investigations in wind tunnel and in flight by the Lockheed Aircraft Corporation to determine whether drag reductions could be obtained at level flight speeds by removing the boundary layer from the wing by means of suction slots. Earlier investigations, by others, indicated that large drag reductions at high lift coefficients with flaps deflected were available, but the use of boundary-layer control at low lift coefficients did not afford improvements in drag, probably because the internal losses were large in the elementary slot-duct entries employed. Because of the recognition of the importance of the internal loss in the slots and ducts, an attempt was made in this investigation to develop satisfactory internal systems, after which the wing was tested in the wind tunnel with suction applied. Finally, a series of tests was made at full scale and high Reynolds numbers in flight when an opportunity arose to apply boundary-layer control to an envelope airfoil that was employed in a low-drag airfoil investigation. The results of this investigation indicated that slot and duct results obtained without flow over the wing may not be applicable directly to flight. Under the turbulent boundary-layer conditions encountered in these tests, the use of boundary-layer control in flight and in the wind tunnel resulted in large wake-drag reductions, but if the high internal losses can be reduced to values obtained without flow over the wing, appreciable net drag reductions can be shown.

H. Julian Allen, USA

Aeroelasticity (Flutter, Divergence, etc.)

345. Harry L. Runyan and Charles E. Watkins, Flutter of a uniform wing with an arbitrarily placed mass according to a differential-equation analysis and a comparison with experiment, Nat. adv. Comm. Aero. tech. Note no. 1848, 50 pp. (Mar. 1949).

This report presents a method for computing the flutter speed of a uniform wing carrying an arbitrarily placed concentrated mass. The analysis is based on the solution of the differential equations of motion at the flutter speed by operational methods and does not require the assumption of specific normal modes of vibration. The procedure is long and tedious but could be organized as a routine for computers. It is an extension of previous work by Goland and Luke which was limited to a mass concentrated at the wing tip. Sample calculations are given in the paper.

The accuracy of the method is then checked by comparing its results with experimental flutter model data where a heavy weight was mounted with its center of gravity near the airfoil leading edge. Very satisfactory correlations between the theory and the experiments are obtained. In contrast, it is demonstrated that the use of the two fundamental normal modes of vibration would lead to incorrect results, although the magnitude and direction of the errors in predicting the flutter speeds by the mode method are not shown. The authors point out the need for considering the higher natural modes of vibration in flutter analyses, using the mode method in those cases where large discontinuities in the mass couplings exist.

Benjamin Smilg, USA

Propellers, Fans, Turbines, Pumps, etc.

(See also Revs. 289, 290)

346. L. C. Burrill, Developments in reversible propellers, Engineering 167, 237–240 (Mar. 1949).

The article is of special interest to marine engineers engaged in the problem of gas turbines for ship propulsion. The author makes a statement of the advantages and disadvantages in fitting a controllable-pitch marine propeller and examines the types of vessels for which it is most advantageous. Several problems in changing the pitch of the blades are investigated, and it is also seen that a good deal of research work has to be done, both in wind tunnels and in cavitation tunnels, to obtain the best type of blade sections for controllable-pitch propellers. The turning moments are given a special examination.

P. N. Brandt-Møller, Denmark

347. Maurice Roy, Pierre Duban, and Georges Gorge, On double flow turbojets (in French), Off. nat. Étud. Rech. aéro. Rep. no. 14, 53 pp. (1948).

This is a detailed performance analysis of gas-turbine power plants of the ducted-fan type. Schemes with and without mixing of the primary and secondary flows are considered.

Values of thrust coefficient and over-all efficiencies based on present and future component characteristics are presented for a series of altitudes and subsonic flight speeds using several mass-flow ratios. It is apparent from the presented data that in order to compare favorably with simple turbojets, ducted-fan engines have to be designed for rather high values of the mass-flow ratio and that the effectiveness of this power plant is limited to a narrow range of air speeds. The suitability of the ducted fan for operation with tailpipe burning is also demonstrated. An increase in thrust up to 400% of the value without tailpipe burning is found to be feasible.

No appreciable difference in performance was found between the schemes using separate passages for the primary and secondary flow and those permitting mixing of the two.

Andrew Fejer, USA

348. A. A. Fulton, Water turbine machinery, Trans. Instn. Engrs. Shipb. Scot. 92, 206–236 (Feb. 1949).

This is a short article about present engineering practice and the fundamentals of selection of water turbines and auxiliaries. The range of heads and specific speeds for applications of Pelton, Francis and Kaplan turbines is discussed, and a brief description of regulation and valve problems is given. R. G. Folsom, USA

349. R. Szewalksi, The overloading of steam turbines by means of additional first-stage nozzle control (in Polish), Arch. Mech. Stos. 1, 120–142 (1949).

The author derives the relation between the increase of normal pressure drop in the control stage and the increase of steam flow, which is approximately equal to the increase of overload range. The criteria of overload range are given. Contrary to current views, the author's numerical results show that the overload range can be substantially increased by the use of additional vents in the control stage. The author is conscious of the difficulties associated with such a measure, and the loss of efficiency accompanying such an overload. A. Uklański, Poland

Flow and Flight Test Techniques

(See also Revs. 290, 314, 317, 339)

350. D. Giles, N. P. L. Mach number gauge, Rep. Mem. aero. Res. Counc. Lond. no. 2131, 4 pp. (1945, publ. 1947).

The device described is used to measure directly the Mach number in an atmospheric-intake-type wind tunnel by applying to a beam balance a moment which is proportional to the difference between the working-section pressure and the intake pressure. This moment is balanced by a weight hung from the beam. The weight required to balance the beam is proportional to the applied moment which, in turn, is proportional to the Mach number. In addition to the calibrated weights, a sliding vernier weight is used for precise balancing. Provision has been made for variations in atmospheric pressure. The point of precise balancing is determined electrically.

John T. Kirkland, USA

351. Paul Guienne, Static pressure orifices insensitive to incidence (in French), Off. nat. Étud. Rech. aéro. Rep. no. 26, 21 pp. (1949).

The author describes a new type of meter for measuring static pressure close to a wing where the flow is approximately two-dimensional. It offers the following advantages: one per cent accuracy when the meter is parallel to velocity; independence of the angle between axis of meter and flow direction within ± 10 deg; very small dimensions; possibility of using the meter within 1 mm of the wing; negligible interference. The type of apparatus which has given the best results consists of a 12-deg dihedral static meter with two holes at a distance of 1 diam from the leading edge. Carlo Ferrari, Italy

352. Walter G. Vincenti and Donald J. Graham, The effect of wall interference upon the aerodynamic characteristics of an airfoil spanning a closed-throat circular wind tunnel, Nat. adv. Comm. Aero. Rep. no. 849, 22 pp. (1946, issued in 1948).

The investigations of Allen and Vincenti (same source, no. 782) on interference between a "two-dimensional" airfoil (chord c) and a rectangular tunnel wall are extended to a circular tunnel (radius r). The present problem, which cannot be treated by the method of images, requires a considerable amount of mathematics, since the representation of the velocity potential due to a horseshoe vortex with infinitesimal span, trailing parallel to the axis of the circular tunnel, is needed and involves a double series in Bessel functions.

In analyzing the effect of the camber, first the downwash at the tunnel axis, $v_z(x)$, is represented by a double integral involving the vorticity distribution (v.d.) over the airfoil. Since it is too difficult to determine the v.d. by the usual method of thin wing theory, one assumes the v.d. as independent of the span station; this assumption is well supported by actual section-lift measurements carried out by the authors. In this way the integration in cross direction can be performed. On developing the remaining integrand up to the first power of c/r and applying standard compressibility corrections, $v_z(x)$ is obtained in the same form as for a rectangular tunnel so that the interference corrections can be expressed by reference to an equivalent (camber) height of a rectangular tunnel.

The thickness effect is determined similarly, the base profile of the airfoil in zero lift position being replaced by a line of vortex doublets along the span. This permits one to start from the same potential formula as before. The resulting $v_x(x)$, corrected for compressibility, can again be referred to an equivalent (thickness) height of a rectangular tunnel.

An independent computation of the wake effect seems unwarranted, and it is proposed to use the v_x (x) due to wake in a rectangular tunnel of equivalent (thickness) height.

Final results are given as a set of correction formulas which give the free air values of V, q, M, R, α, c_l, $C_{m\,c/4}$, c_d as functions of the values of these quantities as found in the tunnel, of $c/(2r)$, and of a thickness variable Λ which is tabulated for some profiles. The choking conditions are also discussed. G. Kuerti, USA

353. C. B. Smith, Wind-tunnel wall corrections for wing-body combinations, J. aero. Sci. 16, 237–242 (Apr. 1949).

This clear paper is a notable contribution to the theory of wind-tunnel wall interference since it treats a subject heretofore neglected and one which appears to be of some importance in interpreting wind-tunnel data. The author has used a rather novel approach to the usual development of tunnel-wall interference on a lifting wing in a circular test section. He then uses the concepts so obtained to derive the additional tunnel-wall interference when a body of circular cross section is present.

The system satisfying the condition of no flow across the tunnel wall for a lifting wing alone is an image vortex of each trailing wing-tip vortex located at the inverse point with respect to the

tunnel wall. The conditions which must be satisfied at the body of a wing-body combination in free air are the same as those of the tunnel wall, only now the image vortex must be inside the body. When the wing-body combination is arranged in the tunnel the requirements at the body are no longer satisfied due to the presence of the wing-tip image vortex, and thus an additional vortex is required inside the body. It is primarily this vortex which modifies the upwash on the wing, and thus the additional tunnel-wall interference when a body is present is largest near the wing root and becomes less in the spanwise direction. The complete equations for the tunnel-wall interference are derived, and a discussion indicating that the series is convergent is presented. The results show that the upwash due to wall interference is doubled at the root when the body is present. It appears, therefore, that for conventional type airplanes this effect would be quite important when interpreting the wing stall patterns. For models with much larger bodies (such as missiles) the effects would be felt over a greater portion of the wing and thus the over-all effect on lift-curve slope would be important.

Henry H. Hoadley, USA

354. Stanley Corrsin, Extended applications of the hot-wire anemometer, Nat. adv. Comm. Aero. tech. Note no. 1864, 62 pp. (Apr. 1949).

New applications of the hot-wire anemometer are proposed and response equations and measuring procedures are developed. One leads to a measurement of significant statistical quantities such as turbulence levels, temperature-fluctuation levels, turbulent heat-transfer coefficient, velocity and temperature scales, and spectra in a turbulent flow with heat transfer. The other leads to measurements in turbulent isothermal mixing of two gases. Direct measure of mean velocity and mean concentrations and the several statistical functions of the fluctuating velocities and local concentrations may be made if the thermal conductivity of the mixture is a known monotone function of the relative concentration; otherwise additional measurements are required. The details of the analysis are worked out on the hypothesis that King's equations for the rate of heat loss from fine wires are accurate, but the method would be equally applicable on other equations of the heat-loss rate. For correlation between different points of spectra and differential measurements the following devices are used: two single bars parallel to each other and normal to the general flow, symmetrically inclined single wires or pairs of double wires, wire pairs of equal or unequal diameters. Errors inherent in the new applications are similar to those experienced in the single anemometer. New error sources are accumulations from required extended arithmetic operations and from the increased time required to obtain aerodynamic and electrical steady-state conditions. The above methods do not appear to be applicable to combustion gas because of excessive temperatures likely to be encountered and the presence of two gas components. Max G. Scherberg, USA

355. A. Martinot-Lagarde, Tail spin (an example of comparison of an airplane with a scale model) (in French), Ingenieur 's-Gravenhage 61, no. 5, L7-L11 (Feb. 1949).

Free-spin experiments that were carried out in the 2-meter vertical wind tunnel of the University of Lille, France, are described in general terms. After a brief discussion of conditions of similitude, of model construction, and of the reduction of the experimental results, the results obtained in the wind tunnel are compared with results obtained in spin tests in actual flight. It seems possible that there exist in flight several spinning states of which only one is realizable in the wind tunnel.

G. Kuerti, USA

356. Georges Monnot, On the measurement of the gas temperature in a Diesel engine (in French), C. R. Acad. Sci. Paris 228, no. 11, 906–908 (Mar. 1949).

An optical method for the measurement of instantaneous temperatures in the combustion chamber of a Diesel engine is described. The radiation through a quartz window is separated by means of a semitransparent mirror into two beams. The transmitted and reflected beams traverse respectively a blue and a red filter and each beam impinges upon a photo-multiplier tube connected through an electronic commutator to the vertical deflection plates of an oscilloscope. Through the use of the commutator, the records of the radiation intensities in the two spectral regions are superimposed on the oscilloscope screen. The ratio of the two intensities can be related to the temperature by Planck's law. In practice, the apparatus is calibrated by means of a pyrometer. Stuart R. Brinkley, Jr., USA

Thermodynamics

(See also Revs. 323, 324, 325, 326, 328, 356, 360, 364, 367, 371, 381)

357. Ronald W. Gurney, Introduction to statistical mechanics, McGraw-Hill Book Co., New York, 1949, 261 pp. Cloth, 9.2 × 6.2 in., 59 figs., $5.

This book is uncommon in many ways. The author has attempted, from the outset, to present the subject of statistical mechanics in an attractive and appealing form. He has succeeded in this attempt to a major extent perhaps because he approached the pedagogical problem in terms of pictures easily grasped by both the student and teacher. The basic problem in statistical mechanics is to determine the distribution of a large number of particles over a set of allowed energy levels for a given value of the total energy. The simple introductory examples in terms of a system of a few particles which are used to illustrate the solution of this problem are clear and well chosen. In addition, a careful distinction is made between information obtained from the results of quantum theory and information obtained from the applications of statistical methods. Finally the relation between statistical mechanics and thermodynamics is presented clearly; such a presentation is in itself uncommon in texts on this subject, since most of these usually leave the student with the impression of having derived the first and second laws of thermodynamics from dynamic principles.

The book is divided roughly into two parts. The first part, comprising the first 7 chapters, gives the reader a novel treatment of the fundamental approach to statistical mechanics without a detailed exposition of the fine points. The second part of the book, comprising the last 5 chapters, retraces the steps in some of the previous chapters, presents the basic principles in greater detail, and includes a discussion of the older approach using classical methods. Throughout most of the book the latest results of quantum theory are used as cornerstones in the applications of statistical mechanics to real systems.

Although it is probable that few teachers in engineering are familiar with the results of quantum theory, they will gain considerably by reading this book. It will increase their understanding, from the microscopic viewpoint, of many of the simple physical processes which are discussed so often in elementary courses in thermodynamics and kinetic theory.

Joseph Kaye, USA

358. J. R. Partington, The foundations of thermodynamics, Phil. Mag. 38, 672–677 (Sept. 1947).

The author does not point out the new ideas in this brief sketch

of an, apparently, conventional treatment of the concepts of work, heat, temperature and entropy.

A. W. Wundheiler, USA

359. W. J. Walker and C. Rogers, The development of variable specific heat charts and graphs, and their application to internal combustion motor problems, Proc. Instn. mech. Engrs. 159, no. 42, 301–312 (1948).

For a given working fluid ($R = 53$ ft/F $= 29$ m/C) with linear variable specific heat the authors have drawn a diagram which makes it possible to read the final temperatures for adiabatic changes of state given by different volume or pressure ratios. In other graphs the heat consumption and the thermal efficiencies are plotted against the compression ratio for various types of indicator diagrams for constant volume, Diesel and dual-combustion cycles.

It would be of course possible to solve some of the problems involved in a more general way by means of the well-known Stodola TS diagram for imperfect gases or a suitable nomographic method. Otakar Maštovský, Czechoslovakia

Heat Transfer; Diffusion

(See also Revs. 289, 328)

360. L. I. Rubinshteĭn, On the initial velocity of the front of crystallization in the one-dimensional problem of Stefan (in Russian), Doklady Akad. Nauk SSSR 62, no. 6, 753–756 (Oct. 21, 1948).

The author's solution of Stefan's problem [Doklady Akad. Nauk SSSR 58, 217–220 (1947)] is examined as $t \to 0$. When solid and liquid have nonzero temperatures at the interface (front of crystallization), the flux of heat there and the velocity of the interface have the form $v = kt^{-1/2}[1 + v^*(t)]$, where $v^*(+0) = 0$. An evaluation of the constant k is given. Corresponding results obtained for cases of zero interface temperature are more complicated.

Courtesy of Mathematical Reviews R. E. Gaskell, USA

361. B. Stuke, Calculation of heat exchange in regenerators with cylindrical and spherical filling material (in German), Angew. Chem. B 20, 262–268 (Oct. 1948).

H. Hausen has developed the theory of heat exchange in gas regenerators in which two gas streams of different temperature periodically flow around storage bodies. His calculations are carried out for regenerators with plates as filling or heat-storage masses. In practice one uses as storage masses not only plates but also bodies with a spherical or cylindrical shape. It is important to know how the results of Hausen's calculations are influenced by the geometry of the storage mass. The author develops equations for the temperature distribution about the cross section of spherical and cylindrical bodies (as limiting forms) and for the coefficient of heat transmission.

The calculations show that the centers of spheres and cylinders are subject to stronger temperature variations than the plate centers. The final equation for the heat transmission coefficient has a similar form for all three geometries of the storage bodies; the remaining differences are expressed by an additional function. Finally a formula for the equivalent thickness of plate yields, with good approximation, one single equation for plate, sphere and cylinder, which allows to calculate the heat-transmission coefficient in all these cases. Walter Fritz, Germany

362. F. C. Becker and N. F. Bisgaard, Analysis of technical problems involved in heating of buildings (in English), Trans. Dan. Acad. Tech. Sci. no. 4, 1–125 (1947).

This booklet contains eight papers from the Laboratory of Heating and Ventilation at the Technical University of Denmark. The fundamental principles of heat transfer are discussed with applications to the calculation of heat losses from buildings. The principles of the construction and use of direction thermometers are discussed. The method of finite differences is applied to problems in unsteady state, heat flow through plane and circular-cylindrical walls, and for period-steady-temperature fields. The use and applications of the Heaviside operational calculus to heat transfer are explained. Calculations are extended to the case of the heat loss from a room under day and night steady-temperature conditions. A. D. Kafadar, USA

363. A. Watzinger and A. Lydersen, Heat transfer for liquids in incrusted tubes on cooling and on heating, Acta Polyt. no. 6, 1–40 (1947).

Forced-convection heat-transfer data are presented for heating and cooling water and 20% sodium-chloride brine in an incrusted circular pipe. The Reynolds-number range extends from laminar through transition to turbulent. The Prandtl-number range (mean temperature range -15 C to $+26$ C) is 8–33. The mean temperature difference ranges from 3 C to 12 C. The incrustation roughness is deduced to be comparable to a Nikuradse sand roughness (ratio of pipe radius to sand diameter) of 60–70 and has been chosen to simulate conditions in condensers and cold-storage heat exchangers. The length-diameter ratio of the pipe is 147. Comparison is made with data in a cleaned pipe. Pressure-drop measurements are included. W. G. Cornell, USA

364. R. F. X. Ruckli, Heat-flow towards the floor of cold stores, situated in the ground level, and calculation of the insulation or the heating system to prevent frost penetration in the ground (in English), Proc. int. Conf. Soil Mech. Found. Engng. 1, 18–22 (June 1948).

In cold stores frost penetrates from the floor into the ground causing ground ice segregation and the uplift of the superstructure. This damage may be avoided by insulation or heating of the floor. The paper proposes a new method of calculating the heat flow through the floor without assuming the ground temperature to be constant at a fixed depth. In the first part there are treated cold stores with circular areas, and in the second part cold stores with infinitely long rectangular areas. The analogy is used between the ground water flow in an artesian well with a plane bottom and the heat flow through the circular cold store situated on the ground level; certain simplifications in the calculation had to be made. The solution of the problem of the artesian well had been given by Forchheimer; this solution is also accepted for the cold store problem. The author develops the formulas for heat flow and temperature distribution; he also calculates the thickness of the insulation and the heating energy needed to prevent the frost penetration into the ground for a given basement construction. Walter Fritz, Germany

365. Charles W. Frick, Jr. and George B. McCullough, A method for determining the rate of heat transfer from a wing or streamline body, Nat. adv. Comm. Aero. Rep. no. 830 (1945, issued in 1948).

The authors develop a method for calculating the rate of heat transfer from the surface of an airfoil or a streamlined body. The transfer of heat from these surfaces to the air stream is a problem of the boundary layer. As a basis, the authors use the investigations of Pohlhausen and Dryden. The results of the theoretical calculation are checked by a number of heat-transfer measurements on a heated wing model in a wind tunnel. The model was heated by a number of incandescent lamps; the

temperatures in the air stream were measured with resistance thermometers and on the model with 41 thermocouples. The results are given in diagrams and tables; we find the chordwise distribution of heat-transfer-temperature difference and the heat-transfer coefficient for laminar boundary layer and with transition to turbulent flow. Further tests were made with regard to attaining the total rate of heat flow from a wing. The experimental study showed that the accuracy of the method for determining the rate of heat transfer is good within the accuracy of the investigation, although the skin temperature does not remain constant along the surface, as is assumed. The total rate of heat transfer is not essentially impaired by such a temperature variation.

Walter Fritz, Germany

366. J. C. Mouzon and C. A. Dyer, Low temperature radiation pyrometry in industry, J. opt. Soc. Amer. 39, 203–210 (Mar. 1949).

The problems encountered in the application of radiation pyrometry in industry are discussed with particular emphasis on the measurement of low-surface temperatures (100 − 400 F). A good review is given of the factors entering into the design and use of radiation pyrometers for such applications.

It is important to note that after consideration of the new methods developed recently for radiation detection, the older and slower responding thermopile was selected. The requirements of mechanical stability and freedom from errors due to poor handling place severe limitation on the methods available for measurement of long wave-length radiation; this point is not made often but is of great importance in almost all radiation problems arising in engineering practice.

Robert Bromberg, USA

367. R. D. Present and A. J. DeBethune, Separation of a gas mixture flowing through a long tube at low pressure, Phys. Rev. 75, 1050–1057 (April 1, 1949).

The separation of a binary gas mixture by diffusion through a long capillary of radius r depends on the fact that the molecules have different masses m_i and mean speeds $\bar{v}_i$. When the inlet pressure is so low that the mean free path λ is much greater than r, the flow is diffusive and the separation factor (at zero outlet pressure) has its maximum value $(m_2/m_1)^{1/2}$. In this limit the flow obeys the Knudsen formula, according to which the number of molecules flowing through a unit cross-sectional area in a unit time is proportional to the pressure gradient.

At high mean pressures ($\lambda \ll r$) no separation occurs; the flow follows the Poiseuille formula, according to which the mean flow is proportional to $p(dp/dx)$. In this limit encounters between pairs of molecules are relatively more frequent than those between molecules and the tube wall. The transport velocities of both kinds of molecules are the same, each bimolecular encounter tending to drag the heavier molecules forward at the same speed as the lighter ones.

This paper treats the intermediate case ($\lambda \gg r$), where the transfer of forward momentum from light to heavy molecules in unlike collisions tends to equalize the transport velocities and to decrease the separation factor. In agreement with experiments on low-pressure flow through porous media, the theory shows that as the inlet pressure rises, the transport of forward momentum from lighter to heavier molecules makes the flow nonseparative before it becomes viscous.

Differential equations for the pressure and composition as a function of distance along a tube are derived by equating the momentum acquired by the light component from the pressure gradient to the momentum lost to the wall plus that transferred to the other component through bimolecular collisions. The viscous effects are treated as a small additive perturbation to the flow. With certain approximations the solution can be obtained in terms of confluent hypergeometric series, though numerical methods of solution are more useful. Simplified expressions applicable to mixtures of isotopes having nearly the same molecular weight are given; at zero back pressure the ratio, Z, of the actual separation efficiency (separation factor minus one) to the maximum value becomes 0.5 when the fore pressure, $p_{50} = 0.978\ \eta\bar{v}/r$. For other fore pressures within the range $0<(p/p_{50})<2$, $Z^{-1} = 1 + 0.662\ (p/p_{50}) + 0.338\ (p/p_{50})^2$.

The differential equations for total flow of a gas mixture are also given.

R. L. Pigford, USA

368. H. Hausen, The influence of Lewis' coefficient on separation of vapors of freezing in gas-vapor mixtures (in German), Angew. Chem. B 20, 177–182 (July 1948).

If a binary mixture of gases or vapors is cooled below the freezing point of one of the components, this gaseous component may condense to a solid state, either as a spongy layer of hoar frost at the cooling surface or partly as fog or snow in the gas stream itself. The author first shows theoretically that the Lewis coefficient ϵ allows to predict which of these types of condensation is more probable and under what conditions it may occur (layer of hoar frost at the surface of the pipe $\epsilon \leq 1$; fog or snow in the gas stream $\epsilon > 1$). The Lewis coefficient is $\epsilon = \alpha/\sigma\gamma C_p$ (α heat-transfer coefficient, σ coefficient of exchange of substance, γ density, C_p specific heat at constant pressure).

In the second part of his investigation the author tries to determine how large ϵ may be for various flows (laminar, turbulent) and gas mixtures. He finds that with laminar flow ϵ may be equal to a/D (a temperature conductivity, D diffusion coefficient). This rate reaches extremely high values in mixtures with hydrogen. With turbulent flow, ϵ suddenly becomes less and reaches the limiting value 1. Experiments with the following mixtures: benzol-hydrogen, water vapor-hydrogen and water-air, show that the theoretical calculations are sufficiently accurate.

Walter Fritz, Germany

Acoustics

(See also Rev. 268)

369. E. Bauer, A theory of ultrasonic absorption in unassociated liquids, Proc. phys. Soc. Lond. Sect. A, 62, 141–154 (Mar. 1, 1949).

The observed absorption of sound in fluids is larger than that predicted by the classical mechanisms of viscosity and heat conduction. The additional absorption is due to the excitation of vibrational and rotational degrees of molecular freedom, as well as to disturbed chemical equilibrium. Calculations based on this assumption have been quite successful for polyatomic gases. In liquids the complication of greater intermolecular forces requires modified treatment to calculate the essential quantity, probability of de-excitation of a forced vibration.

This the author has done for unassociated (low intermolecular force) liquids and has applied the results to computing, by purely thermodynamic reasoning, the absorption coefficients and their temperature coefficients; mean dispersion frequencies; and absorption of binary mixtures. Comparison with rather sparse experimental data indicates that the agreement is within the order of magnitude for the first two effects, while the absorption of sound in a binary mixture of benzene and toluene is predicted surprisingly closely. Future work will attempt to improve the calculation of de-excitation probabilities by using a quasi-crystalline model of a liquid, rather than a modified gas model.

Vincent Salmon, USA

370. Richard K. Cook and Peter Chrzanowski, Absorption by sound-absorbent spheres, J. Res. nat. Bur. Stands. 42, 219–223 (Mar. 1949).

The absorption of a plane wave of sound by a sphere is computed. The calculations are based on the "normal impedance assumption," normal impedance being defined as the complex ratio of sound pressure to the normal component of particle velocity. It was further assumed that this impedance is a constant independent of the direction of incidence. An exact expression including the effects of diffraction has been obtained for the absorption coefficient. The theoretical coefficients were computed for spheres covered with hair felt, using directly measured acoustic impedances. Previously developed statistics were used to apply these calculations to spheres located in a random sound field.

The theoretical coefficients were compared with values obtained experimentally for spheres placed in a reverberation room. Theory and measurements show that these spheres can have air-absorption coefficients appreciably greater than unity. There is, however, a discrepancy between the theoretical and measured values. This discrepancy is assumed to be due to wave propagation through the absorbent material in directions other than normal. Robert Fehr, USA

371. V. P. Peshkov, Conditions for generation and propagation of second sound (in Russian), Zh. exp. teor. Fiz. 18, 857–866 (Oct. 1948).

The laws of hydro- and thermodynamics, combined with certain assumptions suggested by experiment, yield the equations of propagation of first and second sound in helium II. The thermal and filtration methods of second-sound generation, and standing-wave production are discussed. Thermal and viscous wall-absorption losses are estimated. Ed.

372. V. P. Peshkov, Study of properties of the second sound (in Russian), Zh. exp. teor. Fiz. 18, 867–872 (Oct. 1948).

Experiments have demonstrated that second sound is highly stable. The paper's measurements show that the vibration amplitude depends linearly on the power input if the sound is generated by the method of filtration, but this dependence has an anomalous interval of deviation from linearity if the sound generation is thermal. Measurements of second-sound damping in resonators are reported on. Ed.

373. V. P. Peshkov, Velocity of second sound from 1.3 to 1.03 K (in Russian), Zh. exp. teor. Fiz. 18, 951–952 (Oct. 1948).

Measurements in the title range are claimed to corroborate Landau's theory [same source, 11, 592 (1941)] and refute that of Tisza [Phys. Rev. 72, 838 (1947)]. Ed.

374. R. B. Dingle, Second sound and the behaviour of helium II, Proc. phys. Soc. Lond. Sect. A, 62, 154–166 (Mar. 1949).

Theoretical consequences of the existence of second sound in helium II are presented in this short note. A method is indicated for the purpose of setting up a quantitative phenomenological description of thermodynamical behavior of helium II. The paper discusses the variation of the thermal expansion of helium, the ordinary thermal conductivity, the attenuation of second sound and temperature-dependent distributions of energy levels. L. M. Tichvinsky, USA

375. M. A. Isakovich, On sound propagation in emulsions (in Russian), Zh. exp. teor. Fiz. 18, 907–912 (Oct. 1948).

The author gives a theoretical proof of the existence of "thermal" dispersion and absorption of sound in emulsions. The coefficients of absorption depend on the sound frequency. Ed.

376. R. T. Lagemann, D. R. McMillan, Jr., and W. E. Woolf, Temperature variation of ultrasonic velocity in liquids, J. chem. Phys. 15, 369–373 (Apr. 1949).

Measurements have been made of the ultrasonic velocity in 34 organic liquids over a temperature range of 0–60 C by use of a variable-path ultrasonic interferometer operating at 500,000 cps. For all of the liquids studied the relation between velocity and temperature was found to be essentially linear. Values of density, adiabatic compressibility, and temperature coefficient $(\Delta V/\Delta T)$ are also reported. It was found that the successive substitution of a heavier atom in a molecule leads to successively smaller temperature coefficients. It is pointed out that the temperature coefficient of ultrasonic velocity appears to be inversely proportional to the square root of the molecular weight, to a fair degree of accuracy. Authors' summary

Ballistics, Detonics (Explosions)

(See also Rev. 287)

377. R. A. Rankin, The mathematical theory of the motion of rotated and unrotated rockets, Phil. Trans. roy. Soc. Lond. Ser. A, no. 837, 241, 457–585 (Mar. 1949).

The author develops the equations of rocket motion on the basis of the conventional assumption of a nearly perfect rocket in almost-pure-bore motion. The aerodynamic force system is that of Nielsen-Synge [C. L. Nielsen and J. L. Synge, On the motion of a spinning shell, Quart. appl. Math. 4, no. 3, 201–226 (1946)]. Only the motion during burning is treated, and for this it is assumed that the thrust is really symmetric in position and line of action, and outranks the drag and the axial component of gravity.

From the equations of motion, it is shown that each of the variables describing the motion can be expressed as a linear function of the initial (at time of launch) values of those variables, added to a particular integral being a linear combination of eight parts, each corresponding to one of the following eight disturbing factors: gravity, wind, and six "tolerances" associated with the directions of the principal axes of inertia, the charge center of gravity, the rotational torque axis, the thrust-application point, the center of the assumed exit plane of the gases, and the thrust.

Explicit solutions in terms tabulated and their integrals are obtained for the special case where the spin of the rocket is proportional to its velocity. The effects on the perturbed motion of the initial conditions and the eight other disturbing factors are considered individually and in great detail.

These solutions are then simplified and made expressable in terms of Fresnel functions for rapid calculation, by introducing some additional assumptions including constant acceleration and the neglect of certain damping parameters.

Other forms of spin are then considered, in order to show the effect on the motion, particularly on its stability of "overspinning" or "underspinning" the rocket at launch. The results for the case of zero spin agree with those of Rosser, Newton and Gross [*Mathematical theory of Rocket Flight*, McGraw-Hill Book Co., New York, 1947; see Rev. 1, 912], though the author did not know of the book at the time of writing this paper.

Next, a discussion of stability and the various causes of possible instability is given, based on the equations with general spin. It is found that there are three such important causes; the author calls the corresponding effects "muzzle resistability, nonprecessioned resistability, and resonance instability."

The paper closes with a discussion of the various methods of imparting spin, and finally a summary of the results obtained.

Felix Rosenthal, USA

378. M. J. Bouffard, New viewpoint in exterior ballistics (in French), Mémor. Artill. fr. no. 4, 22, 783–835 (1948).

The basic idea of the paper is to consider the more involved hodograph representation of the resistance vector rather than that of the velocity vector. The resistive acceleration vector R is plotted against the flight angle on a polar diagram. The ratio P of the acceleration due to gravity to the intercept on the vertical axis of the tangent to this resistance polar at a point is termed the "ballistic index" at this point.

In the first section of the paper the author investigates the inverse problem of ballistics: the motion of a given trajectory. It is shown that the required ballistic index is related to the trajectory by the relation

$$p = 4 - 2y'' y''''/y'''^2$$

where the differentiations are with respect to the horizontal abscissa x. In particular, it is found that the trajectory of Piton-Bressant, the hyperbola with a vertical asymptote, and the parabola with axis inclined, correspond to a constant value of p.

In the second section the motions with constant ballistic index are investigated comprehensively and determined in terms of the index p and the initial conditions.

The third section treats the approximation at a point of the actual motion of the projectile by one of constant ballistic index. This "surosculatory" motion is one which fits the actual one not only in vectorial velocity and resistance magnitude, but also in the ballistic index, giving a high degree of fit. The index, of course, must be calculated for the actual motion and is expressed as a function of velocity (with the resistance law), the altitude, and the flight angle.

In the last section the author develops numerical methods for calculating trajectories, using modified surosculatory trajectories and devising higher-order approximations for better accuracy.

Wallace D. Hayes, USA

379. J. Ackeret, Elementary discussion of the stability of long projectiles (in German), Helv. phys. Acta 22, no. 2, 127–134 (1949).

The author discusses the stability of a slender spin-stabilized projectile. He uses the simple Munk airship theory and Tsien's linearized theory of the lift of an axially symmetric body in a supersonic flow to estimate the aerodynamic moment (hence the restriction to slender shells). The usual stability criterion for the required spin rate is obtained. H. J. Stewart, USA

380. P. W. Bridgman, The effect of high mechanical stress on certain solid explosives, J. chem. Phys. 15, 311–313 (May 1947).

The detonation of solid explosives is usually initiated by a combination of a large mechanical stress and a high temperature. It is important to know whether either of these factors, acting on its own, is sufficient to cause detonation and, in this paper, the author describes experiments whose aim was to discover whether explosives will undergo detonation at ordinary temperatures when subjected to pressures of the same order as those in a detonation front. The apparatus has already been described [Proc. Am. Acad. 71, 387 (1937)] and here different kinds of explosives were subjected (a) to an average hydrostatic pressure of 50,000 kg/cm², on which was superposed a shearing stress sufficient to produce a drastic shearing deformation of the order of 60 radians, and (b) to an average hydrostatic pressure ranging up to 100,000 kg/cm², but without explicitly applied superposed shear. Eleven explosives were subjected to the first test and, of these, only four detonated. In the second test with four explosives, only one detonated. The author concludes that there was a certain amount of ambiguity in the instances where detonation occurred and that these could be accounted for by secondary effects, such as striking of sparks by fractured fragments of steel, or local elevations of temperature. The general conclusion is that mechanical stresses of the order of those used in the experiments were not, per se, capable of initiating detonation.

R. M. Davies, Wales

Soil Mechanics, Seepage

(See also Rev. 296)

381. Gregory P. Tschebotarioff and Philip P. Brown, Lateral earth pressure as a problem of deformation or of rupture (in English), Proc. Sec. int. Conf. Soil Mech. Found. Engng. 2, 81–86 (1948).

This surprising report on earth-pressure tests, carried out on large-scale models at Princeton University since 1943, will substantially change our concept of earth pressure and, in particular, of Coulomb's theory of rupture and Rankine's earth-pressure theory at rest. The authors measured earth pressure exerted by submerged sand and consolidated clays on anchored elastic bulkheads.

Pressures of cohesive soils were studied first; it was found that the conventional (Coulomb) assumption that there is a zone of zero lateral pressure against the upper portion of the retaining wall does not hold. The large dimensions of the testing tank (10 × 13 × 18 ft) eliminated the effects of friction against the side walls. The sandy clay (plasticity index 7%) was placed in the tank in a semiliquid consistency. The deflections of the flexible wall and the pore pressure decreased continuously during consolidation. Shearing strength tests on undisturbed samples did not confirm the classical earth-pressure theory: the pressure distribution along the back of the wall follows rather the law of a power parabola than a hydrostatic distribution.

Using the moment distribution obtained by measuring bending stresses, shears and lateral-pressure distribution were determined by means of double differentiation; deflections were directly measured. It was found that Krey's method of structural analysis and the principle of the elastically supported beam ($p = Cy$) do not hold, and the theory of earth pressure acting on underground structures has to be revised. Present theories lead to overdesigned walls, real earth pressures being substantially smaller than the computed ones. They are also independent of the deflections of the wall, and never less than the pressure of the so-called "consolidated equilibrium." No relationships could be established between the results of lateral-pressure measurements and the shearing strength. Arching in submerged sand does not reduce the wall pressure; but shearing stresses do between the wall and fill.

The new semiempirical structural analysis is being worked out, and final recommendations cannot be made before completion of the whole research program. J. Jáky, Hungary

382. A. W. Skempton and H. Q. Golder, Practical examples of the $\varphi = 0$ analysis of stability of clays, Proc. Sec. int. Conf. Soil Mech. Found. Engng. 2, 63–70 (1948).

This article first presents, summarily, the conditions under which the analysis, commonly designated the "$\varphi = 0$ method," can be used in stability problems involving clay soils: the soils

should be saturated, stressed under conditions of no-water-content change and have a zero angle of shear resistance. It is emphasized that, under these conditions, the correct position of the shear surface is controlled by the true angle of internal friction. However, the method will lead to a correct estimate of the factor of safety provided that the shear surface used is compatible with the assumption that the clay is nonfrictional.

The article then presents the results obtained by applying the $\varphi = 0$ method to the determination of the factors of safety of about 12 clay-earth masses, both natural and artificial, in which failures were observed. The LL of these earth masses extended from 40 to 145% and the shear resistances from 240 to 650 lb/ft². The figures obtained for the factors of safety, ranging from 0.9 to 1.15, confirmed satisfactorily the value of the $\varphi = 0$ analysis.

Manuel Rocha, Portugal

383. J. Brinch Hansen and R. E. Gibson, Undrained shear strengths of anistropically consolidated clays, Géotechnique Lond. 1, 189–204 (June 1949).

The authors point out that since clays are usually consolidated under effective vertical stresses which differ considerably from effective stresses in the horizontal direction, the shear strength of the soil is dependent on the inclination of the shear plane. Furthermore the effective stresses which determine the shear strength in nearly every saturated normally loaded clay, are different depending on whether the clay has undergone compression, expansion or recompression. This fact is taken into account in the shear strength formula by introducing a compressibility ratio λ. Assuming conditions of constant volume and plane strains, expressions for c/p (c shear resistance mobilized, p vertical effective stress) are developed on planes of different inclination. Three particular cases are then considered and the corresponding formulas given. These are the active and the passive conditions and the condition of failure on a horizontal plane. Presumably more exact expressions than those in common use are also developed for computing c/p from vane tests, unconfined compression tests, quick shear-box tests and consolidated quick triaxial tests.

Eben Vey, USA

384. L. Fox, Computation of traffic stresses in a simple road structure, Rd. res. tech. pap. no. 9 (1948).

An exposition of the Burmister method and Southwell's relaxation method for the computation of stresses in a two-layer system.

Ed.

385. Wilson V. Binger, Analytical studies of Panama Canal slides (in English), Proc. Sec. int. Conf. Soil Mech. Found. Engng. 2, 54–60 (1948).

This article presents the results of stability studies, by the conventional Swedish Circle Method, of two Panama Canal slopes in which important slides have successively occurred.

First, values of the average shear strength of the soil were determined based on the assumption that the factor of safety was 1.0 just prior to failure. These values indicate that the shear strength, owing to the progressive disturbance caused by the slides, decreased, from 1912 to 1947, to about 20% of its initial value.

Second, the values of the cohesion and angle of friction adopted in the recent studies for increasing the capacity and security of the Canal, were used in the determination of the factors of safety of the slopes before the beginning of the slidings. Although the factors are less than unity, showing that these strength assumptions were conservative, the calculated slopes were steeper than those reached in 1947.

Manuel Rocha, Portugal

386. J. L. A. Cuperus and F. C. de Nie, Strengthening the road-bed of a railway, supported by soft soil and situated amidst the buildings of the central part of a town (in English), Proc. Sec. int. Conf. Soil Mech. Found. Engng. 2, 1–7 (1948).

In other to eliminate the continual settling and danger of sliding of the roadbed of a railway embankment, sand was dumped in parallel ditches on both sides of the embankment. This sand was sustained by steel sheet-piling.

The article describes tests previously made, on an experimental section of the embankment, with the purpose of observing displacements and, especially, the changes in the water pressure of the ground pores when the sand load is applied.

Manuel Rocha, Portugal

387. G. Remenieras, The application of electro-osmosis to certain works in water-bearing soils (in French), Houille blanche no. spec. A, 393–404 (1949).

This paper reviews the well-established principles of electro-osmosis as applied to the drainage of fine grained soils. The closely associated electrophoresis and the electromotive force of filtration are discussed briefly. The experiments performed by M. M. Schaad and Haefeli of the École Polytechnique, Zurich, on electro-osmosis applied to soils are discussed at considerable length. The author mentions in conclusion certain tests which he is performing relative to the desilting of water by means of electrophoresis.

Eben Vey, USA

Geophysics, Meteorology, Oceanography

(See also Rev. 326)

388. John H. Hogson, Analysis of travel times from rockbursts at Kirkland Lake, Ontario, Bull. seism. Soc. Amer. 37, 5–17 (Jan. 1947).

The paper can be considered an extension of E. A. Hodgson's original work on wave velocity and crustal structure in the vicinity of Ottawa (Bull. seism. Soc. Amer. 32, 249–255). The rockburst which occurred on December 27, 1938, in Lake Shore Mines, Ontario, was of significant scientific nature, and formed the starting point of a program of seismic research. In order to further study these rockbursts, Dr. Hodgson was consulted and a suitable equipment consisting of a Heiland prospecting-type geophone, and a specially designed 30 mm-per-min paper speed recorder set up. This seismograph remained in operation for nearly six years and recorded some 15 rockbursts.

The present paper describes these rockbursts by listing the travel times of observed phases and deducing time-distance curves and crustal structure from these observations. The profile presented has one end in the Canadian Shield and the other in New England. The author suggests that, since the outbursts are continuing, a unique opportunity is presented to extend our knowledge of the crust, which forms a basic element of studies of gravity, isostasy, thermal gradients, tectonophysics and geology. The subdepartment of geophysics of the University of Toronto forms an ideal group for the special geophysical study of the Canadian Shield.

S. K. Ghaswala, India

389. Myron Tribus, G. V. W. Young and L. M. K. Boelter, Limitations and mathematical basis for predicting aircraft icing characteristics from scale-model studies, Trans. Amer. Soc. mech. Engrs. 70, 977–982 (Nov. 1948).

The authors consider the problem of determining the icing characteristics of bodies of such a complex shape that an analytical solution would be time-consuming or almost impossible. It is shown, by manipulation of the equations of motion of a water drop

impinging on a body, that regardless of the body shape, the solutions will be functions of two dimensionless moduli, the Reynolds modulus of the drop (R_ν) and a scale modulus. Further, it is demonstrated that the *ratio* of these two moduli is the pertinent modulus only when Stokes' law is valid for the drop (i.e. when $R_\nu \leq 1$).

Application of these results to model studies under icing conditions on the ground (i.e. mountain tops and wind tunnels) where the air velocities are, on the average, only a fraction of the cruising speed of aircraft, shows that the model sizes must be *larger* than the prototype in order to maintain similarity. In the past, the trend has been to test models of a size equal to or smaller than the prototype, with the result that application of the model data to the prototype was in error. A. G. Guibert, USA

390. K. S. Ramamurti, Long-term longitudinal oscillation of the atmosphere in the vicinity of South America, J. Met. 6, 63–65 (Feb. 1949).

A regression equation involving four parameters is shown to fit longitudinal profiles of annual departure from normal pressure for stations in South America for all the years from 1902 to 1930. The values of the parameters are shown to be statistically significant measures of the annual changes in circulation patterns.

M. Neiburger, USA

391. S. Mull and Y. P. Rao, Theory of cumulus convection (in English), Indian J. Phys. 22, 531–538 (Dec. 1948).

A qualitative model for the structure of the convection cell is presented in four stages. In the initial stage the isobaric surfaces, assumed horizontal before the convection starts, are bulged upward up to some level, so that below this level the pressure is highest at the center of the perturbation, resulting in outflow. The outflow causes a fall of pressure at the ground, and inflow begins there, producing an up current in the lowest layer. In the "large cumulus stage" the boundary between inflow and outflow has risen due to the continued outflow aloft. With the beginning of the cumulonimbus stage the top portion of the cloud comes up into the high pressure region and the spreading of the cloud top starts. The fully grown cumulonimbus is characterized by the equality of the convergence below and the divergence above. The high pressure has its maximum some distance below the top of the cloud, because of the upper part of the cloud being denser than the surroundings. The entrainment of air into cumulus is thus shown by the authors to be a necessary part of convective circulation. The mechanism of breakdown of the simple convective cell into a thunderstorm is to be discussed by the authors in another paper. M. Neiburger, USA

392. Everett F. Cox, Abnormal audibility zones in long distance propagation through the atmosphere, J. acoust. Soc. Amer. 21, 6–16 (Jan. 1949).

It has been known that the temperatures of the atmosphere at altitudes of about 55 km, where the ozonosphere is located, must be greater than the temperature at the surface of the earth. Sounds which originate at the surface can be reflected back, similarly to the way in which radio waves are reflected back by the ionosphere. The amount of time which elapses between the initiation of a sound pulse at the surface and its return (several hundred kilometers away) can be used to obtain temperatures above the stratosphere.

At the time of the detonation of 5000 tons of explosive on Helgoland in April 1947, the Naval Ordnance Laboratory secured recordings of the sounds of very low frequency produced in the atmosphere and reflected back by the ozonosphere. The recordings were obtained with a microbarograph system consisting of a mutual-inductance microphone, an amplifier, and a pen-on-paper recorder. The system covered the frequency range from 0.05 to 5 cps. Ten recording stations were located south-southeast from Helgoland at distances ranging from 66 to 1000 km. The recordings yielded sound-pressure magnitudes, and elapsed times between the detonation and arrivals of the sound at the stations.

The data were used to deduce the temperature at several altitudes above the stratosphere. For example, the temperature at an altitude of 55 km was found to be 21 C (surface temperature $\neq$ 7 C). This temperature is considerably lower than those found by earlier investigators. The data also showed that waves which were propagated to greater distances had relatively more high-frequency energy. The author's attempt to explain this analytically was subsequently acknowledged to be in error [J. acous. Soc. Am. 21, 501 (Sept. 1949)]. Richard K. Cook, USA

393. Ulrich Roll, The wind field over sea waves (in German), Naturwissenschaften 35, no. 8, 230–234 (1948).

After mentioning previous theories about the mechanism of energy transfer from the wind to water-surface waves, the author discusses thoroughly the experimental results of H. Motzfeld who measured the wind distribution over wavelike rigid surfaces in a wind tunnel. These results are interpreted according to Prandtl's equations for boundary layers over rough surfaces in the case of sharp-crested waves, and according to von Kármán's velocity distribution, over smooth planes in the case of sinusoidal surfaces. The author now presents new experimental data of wind-velocity distribution over a flat water surface with simultaneous recordings of the surface waves. These are compared with Motzfeld's model measurements. It is found that the parameter of roughness depends on the factor $(\tau_0/\rho)^{1/2} = u^*$, the so-called shearing-stress velocity, so that the vertical wind distribution over flat water can be written as $u = 5.75u^* \log 2.1\ u^*/\nu\ (z + h/2)$ (ν, kinematic viscosity, h, height of surface waves). Further conclusions, in agreement with Motzfeld's model measurements, are that in the sea, the energy transfer from wind to water is due more to tangential friction forces than to pressure differences.

Horst Merbt, England

Marine Engineering Problems

(See Rev. 346)